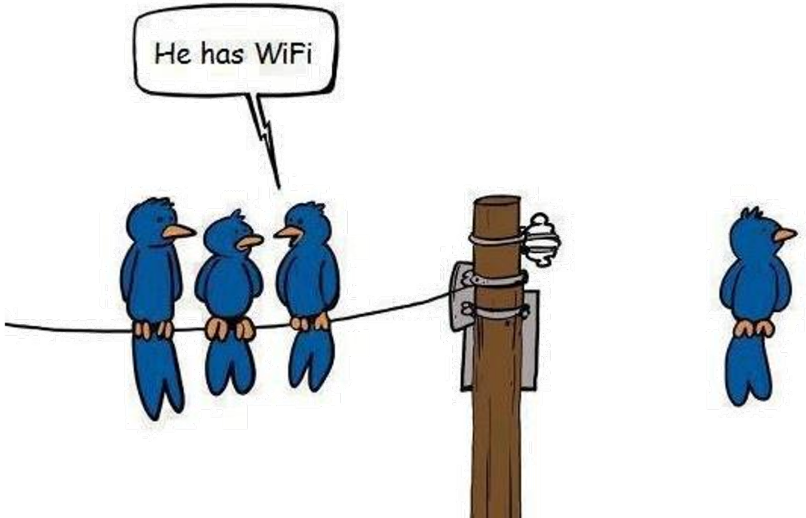
ECE 598HH: Advanced Wireless Networks and Sensing Systems

Lecture 3: Wireless Channel + OFDM Part 1

Haitham Hassanieh



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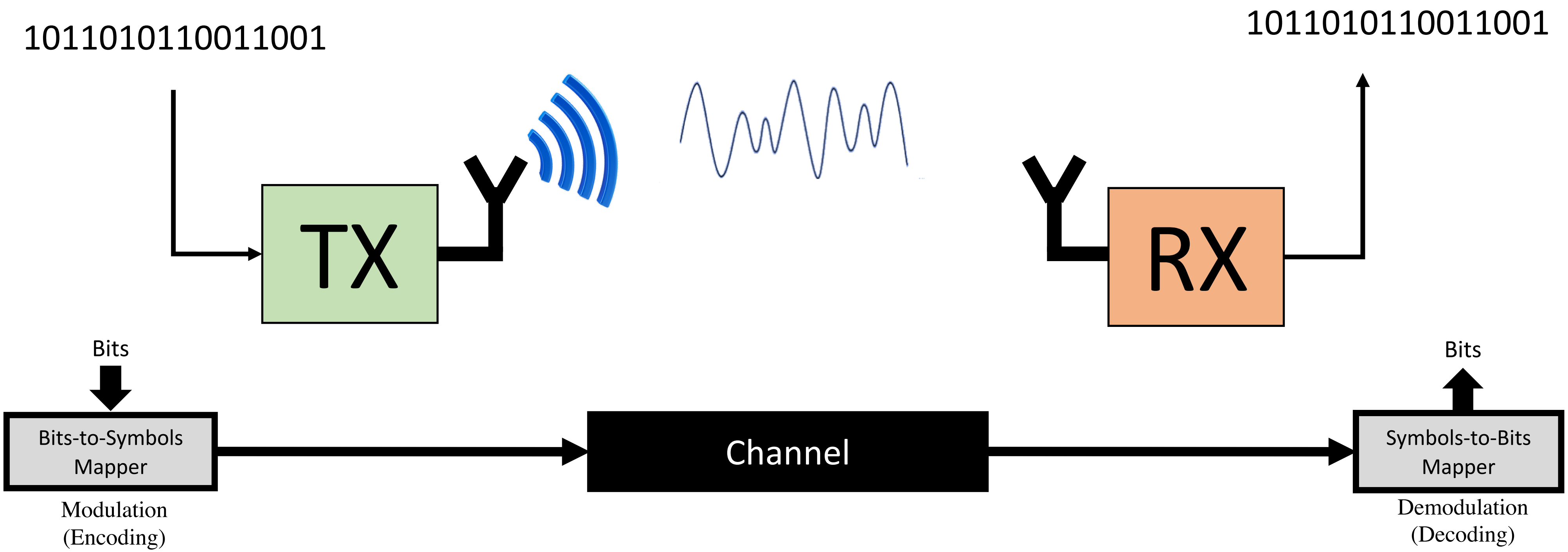
The Channel

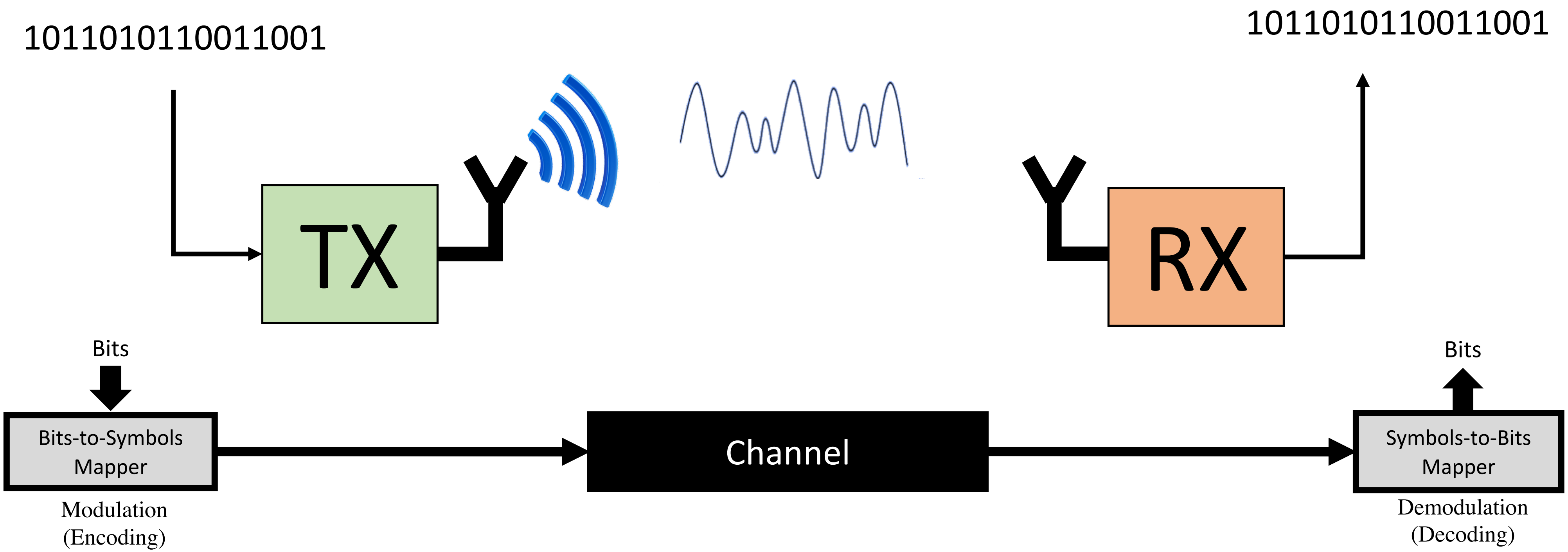


$x(t)$ ➡ ➡ $y(t) = x(t) + v(t)$

Channel adds noise (AWGN)!

$$v(t) \sim N(0, \sigma)$$

The Channel

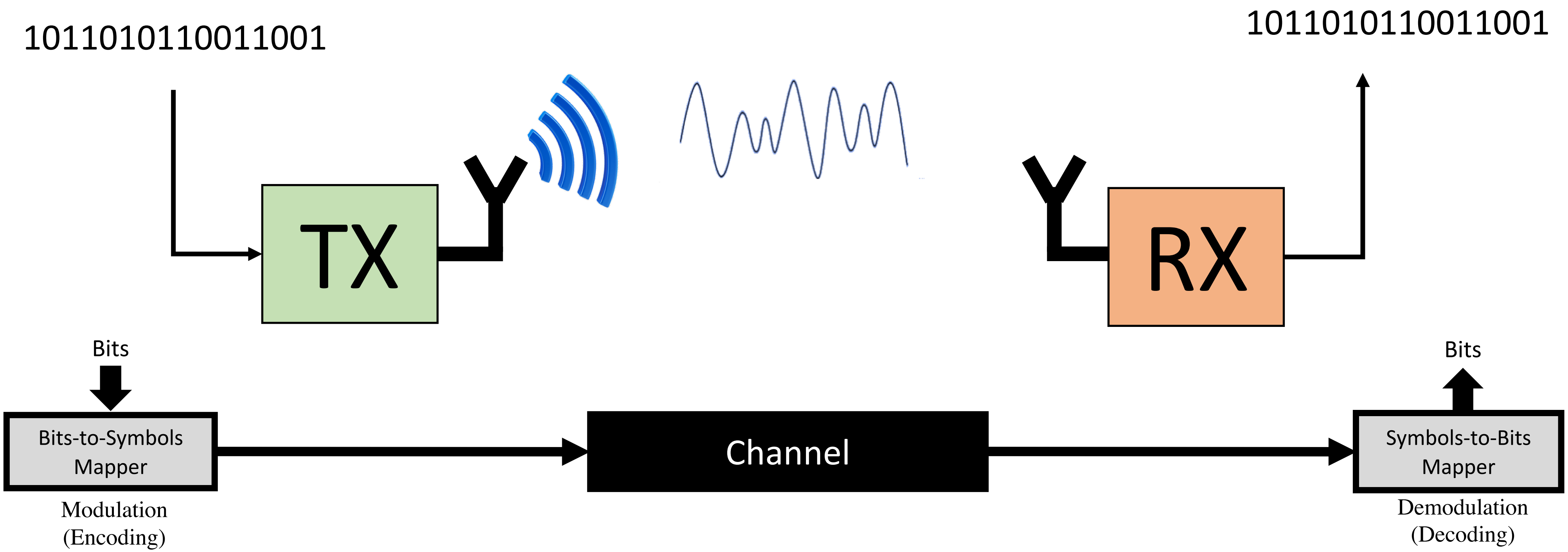


$x(t)$ ➡ ➡ $y(t) = x(t-\tau) + v(t)$

Channel delays the signal!

$$\tau = \frac{d}{c}$$

The Channel

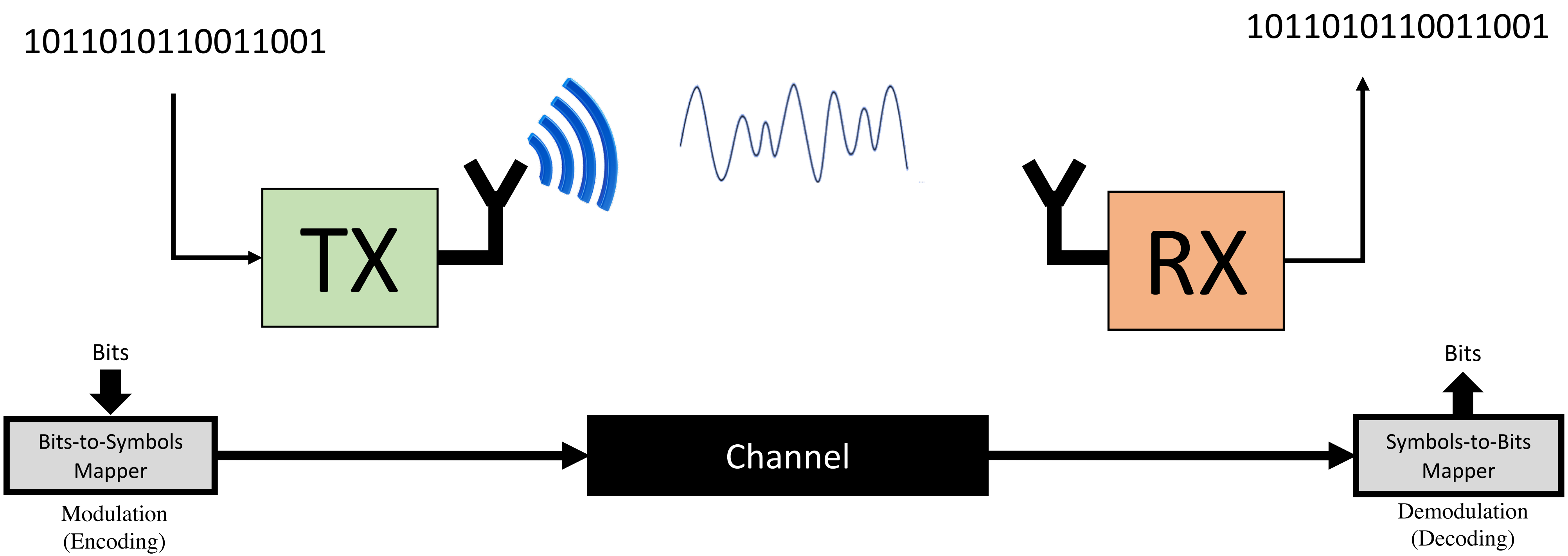


$x(t)$ ➡ ➡ $y(t) = \boldsymbol{h}\, x(t-\tau) + v(t)$

Channel attenuates the signal (Pathloss)

$$P_{RX} = G_{TX} G_{RX} \frac{\lambda^2}{(4\pi d)^2} P_{TX} \quad \Rightarrow \quad |h| \propto \frac{\lambda}{d}$$

The Channel

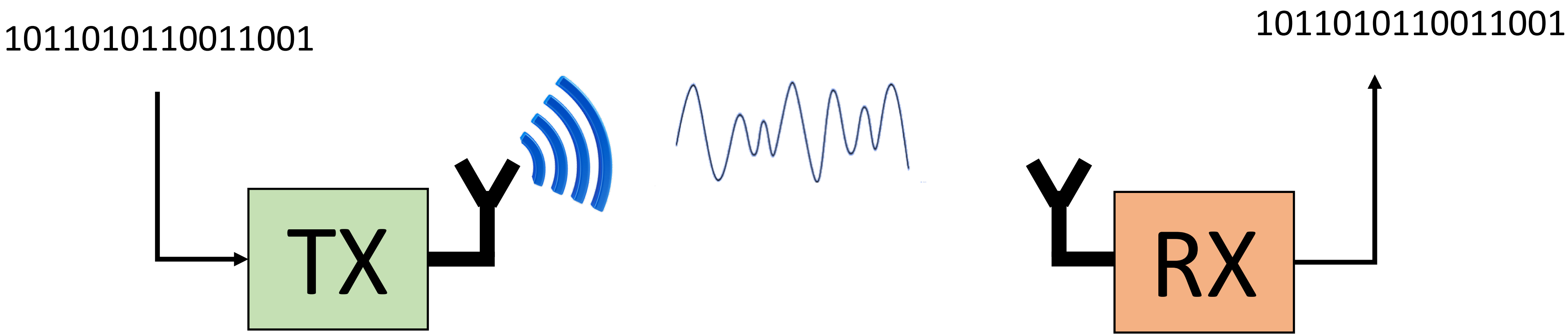


$x(t) \rightarrow$ $\rightarrow y(t) = \mathbf{h}\, x(t-\tau) + v(t)$

Channel rotates the signal (Adds Phase)

$$h \propto \frac{\lambda}{d} e^{j\phi}$$

The Channel



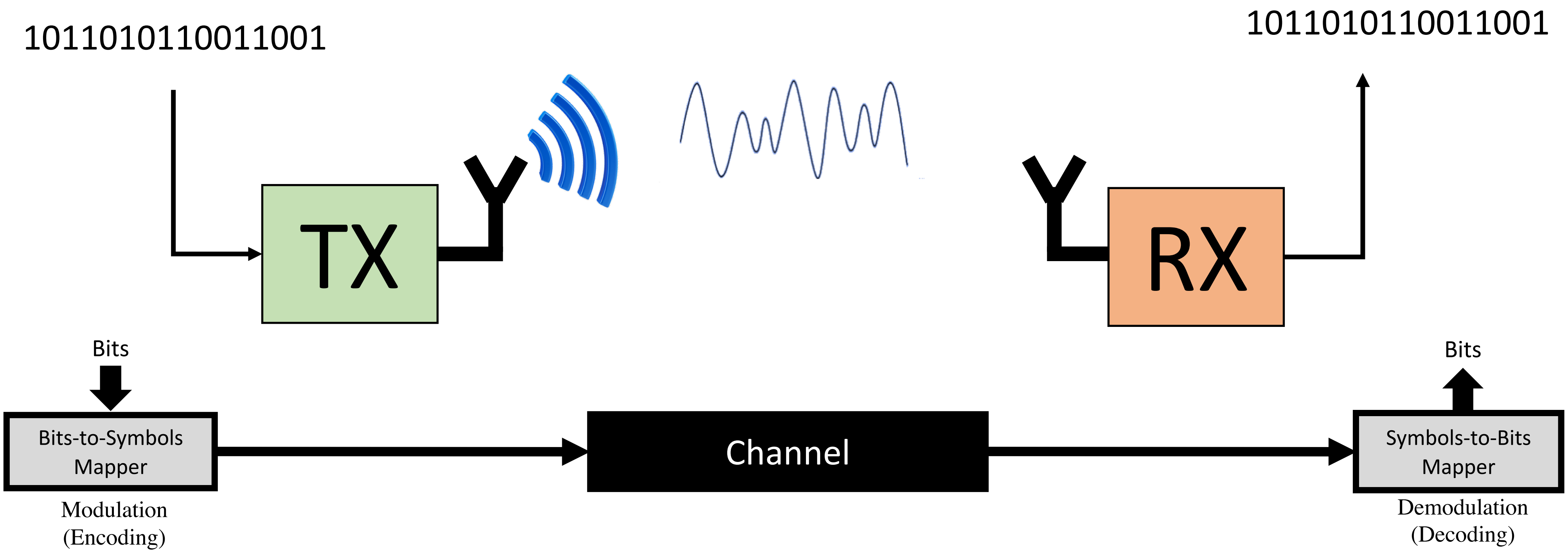
$x(t)$ ➡ ➡ $y(t) = \mathbf{h}\, x(t-\tau) + v(t)$

⬆

$x(t)\ \times e^{-j2\pi f_c t}$ ➡ $|h| x(t-\tau) e^{-j2\pi f_c (t-\tau)}$ ➡ $\times e^{j2\pi f_c t}$ ➡ $|h| x(t-\tau) e^{j2\pi f_c \tau}$

$$h \propto \frac{\lambda}{d} e^{j\phi} \;\rightarrow \phi = 2\pi f_c \tau = 2\pi \frac{c}{\lambda}\frac{d}{c} = 2\pi \frac{d}{\lambda} \;\rightarrow h \propto \frac{\lambda}{d} e^{j2\pi d/\lambda}$$

The Channel



$x(t)$ ➡ ➡ $y(t) = \mathbf{h}\, x(t-\tau) + v(t)$

Channel:

- Adds Noise
- Delays the Signal
- Attenuates the Signal
- Rotates the Phase of the Signal

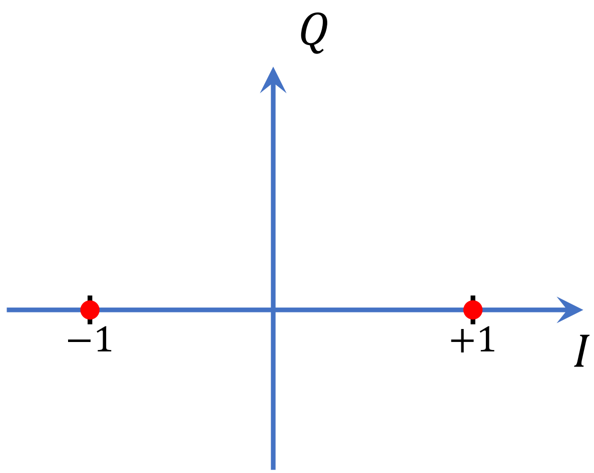
$$h \propto \frac{\lambda}{d} e^{j2\pi d/\lambda}$$

The Channel

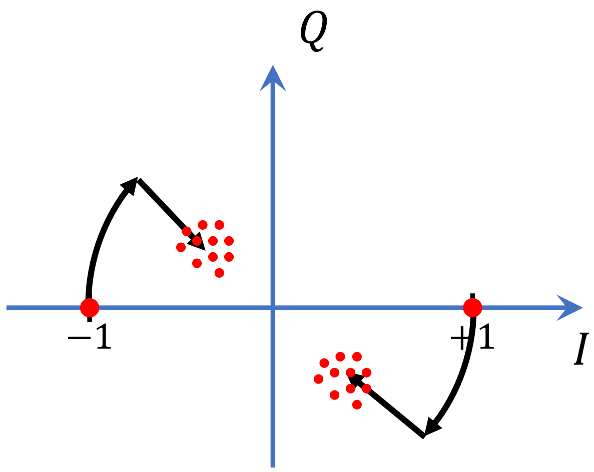
Consider BPSK Modulation.

$$0 \rightarrow -1$$

$$1 \rightarrow +1$$



$x(t)$



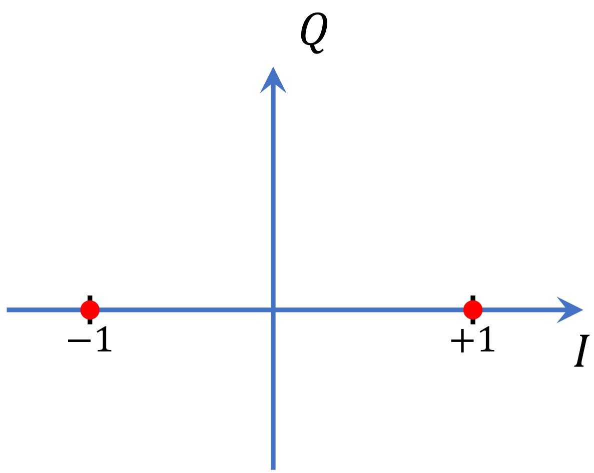
$h \times x(t) + v(t)$

The Channel

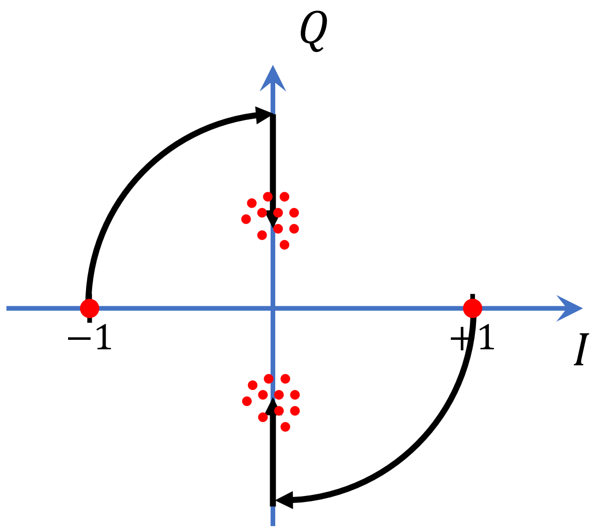
Consider BPSK Modulation.

$$0 \rightarrow -1$$

$$1 \rightarrow +1$$



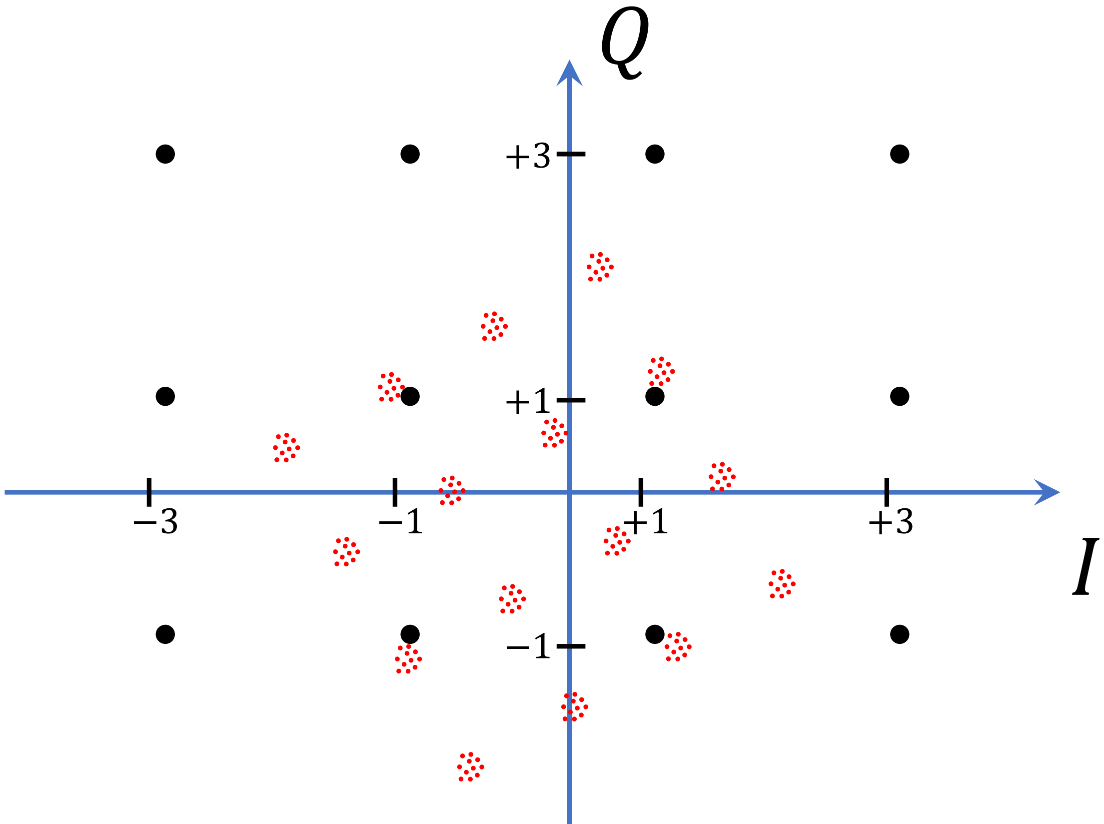
$x(t)$



$h \times x(t) + v(t)$

The Channel

Consider QAM Modulation



Demodulating correctly requires COHERENCE!

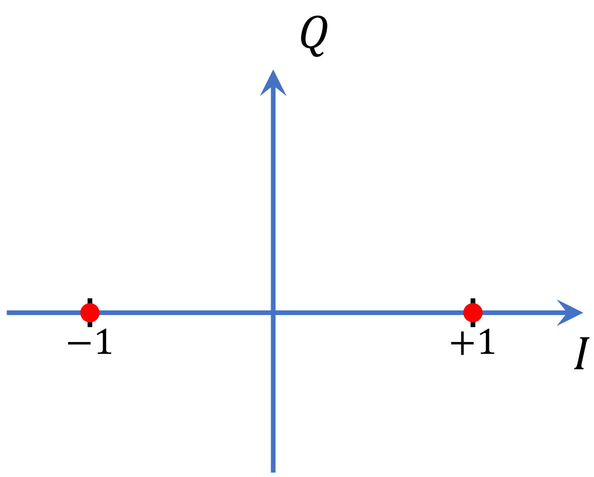
i.e., Need to estimate & correct for the channel h

Channel Estimation & Correction

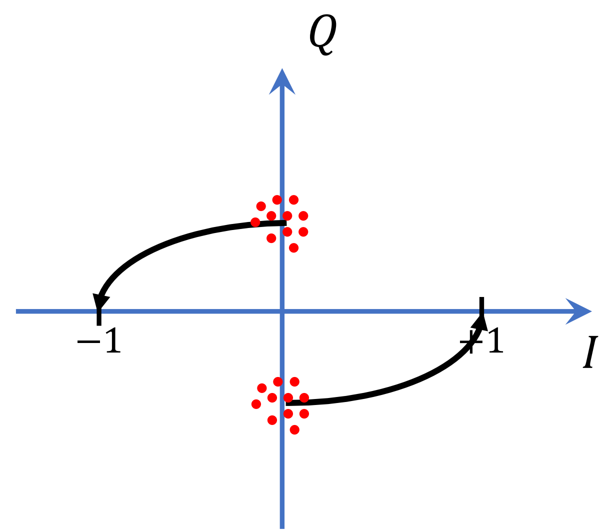
Channel Estimation & Correction

Consider BPSK Modulation.

$$0 \rightarrow -1$$
$$1 \rightarrow +1$$



$x(t)$



$h \times x(t) + v(t)$

Send Training Sequence (Preamble Bits): Known Bits

$x(0) = 1 \longrightarrow y(0) = h + v(0)$

$x(1) = 1 \longrightarrow y(1) = h + v(1)$

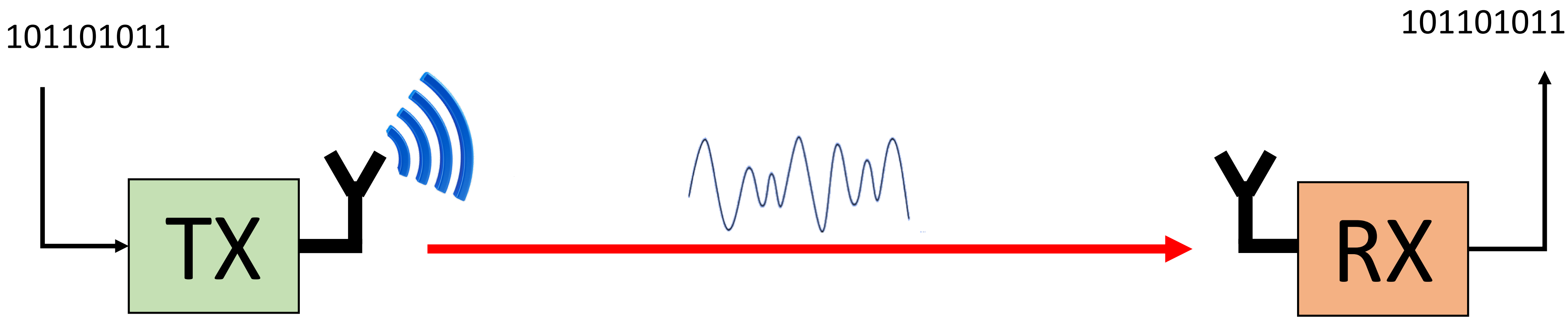
$x(2) = -1 \longrightarrow y(2) = -h + v(2)$

$\vdots$

Estimate channel: $\tilde{h} = \frac{1}{K} \sum_{k=1} \frac{y(k)}{x(k)}$

Correct channel: $\tilde{x}(t) = \frac{y(t)}{\tilde{h}}$

The Channel

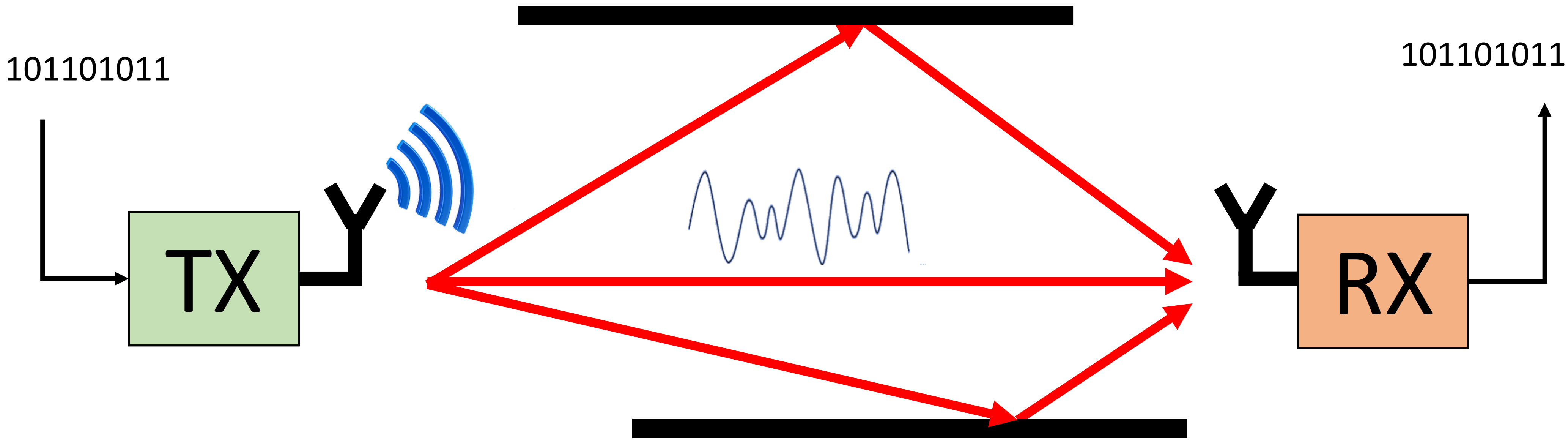


$x(t)$ ➡ **Channel** ➡ $y(t) = \mathbf{h}\, x(t-\tau) + v(t)$

$$h \propto \frac{\lambda}{d} e^{j2\pi d/\lambda}$$

Assumes single path!

Multipath Channel



Multipath Propagation: radio signal reflects off objects ground, arriving at destination at slightly different times

$$y(t) = \alpha_1 e^{\phi_1}\, x(t-\tau_1) + \alpha_2 e^{\phi_2}\, x(t-\tau_2) + \alpha_3 e^{\phi_3}\, x(t-\tau_3) \cdots$$

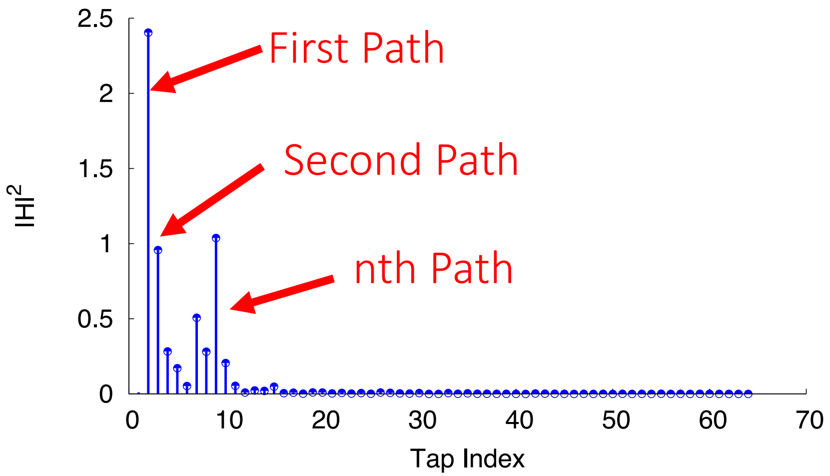
$$y(t) = \sum_k \alpha_k e^{\phi_k}\, x(t-\tau_k) = \sum_k h(\tau_k)\, x(t-\tau_k) = h(t) * x(t)$$

$h(t)$ is channel impulse response.

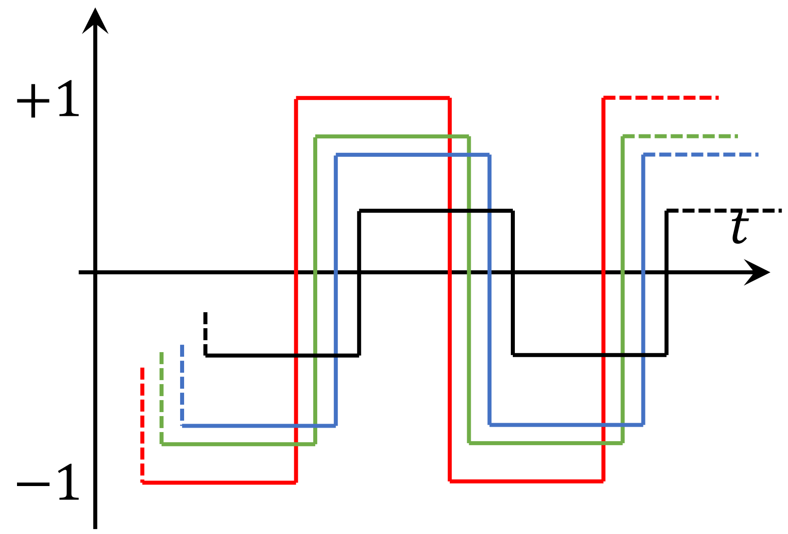
Multipath Channel

$h(t)$ is channel impulse response.

$$y(t) = \sum_{k} h(\tau_k)\, x(t - \tau_k) \;= h(t) * x(t)$$



Multi-tap Channel



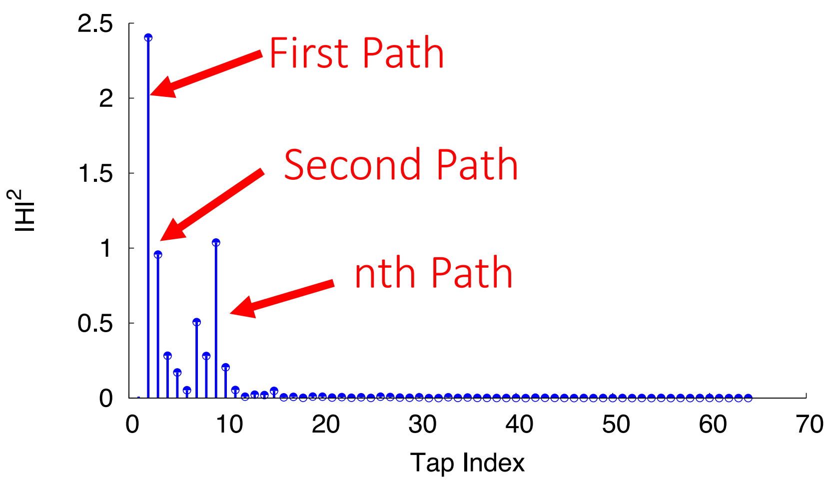
ISI: Inter-Symbol-Interference

Symbols arriving along late paths interfere with following symbols.

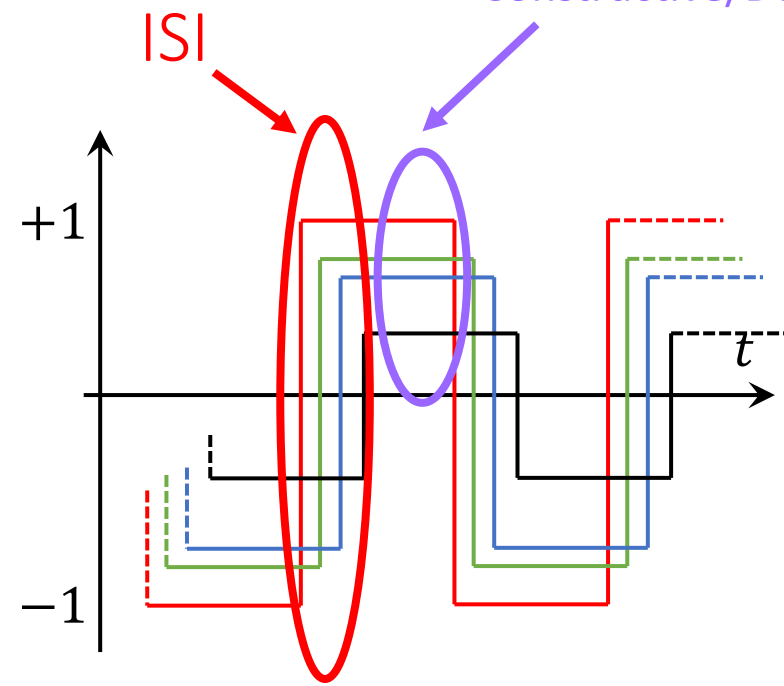
Multipath Channel

$h(t)$ is channel impulse response.

$$y(t) = \sum_k h(\tau_k)\, x(t-\tau_k) \; = h(t) * x(t)$$



Multi-tap Channel



ISI: Inter-Symbol-Interference

Symbols arriving along late paths interfere with following symbols.

Channel Fading

Symbols arriving along different paths sum up destructively

Multipath Channel

$h(t)$ is channel impulse response.

$$y(t) = \sum_k h(\tau_k)\, x(t - \tau_k) \; = h(t) * x(t)$$

Channel Fading: Symbols arriving along different paths sum up destructively

Example 2 paths with distance $d_1 = 1m, d_2 = 1.06m$:

$$h = \; h_1 + h_2 \; = \frac{\lambda}{d_1}\, e^{j2\pi d_1/\lambda} + \frac{\lambda}{d_2}\, e^{j2\pi d_2/\lambda}$$

$$= \frac{\lambda}{d_1}\, e^{j2\pi d_1/\lambda} \left(1 + \frac{d_1}{d_2}\, e^{j2\pi(d_2 - d_1)/\lambda}\right) \qquad \frac{d_1}{d_2} \approx 1$$

$$if\ \frac{d_2 - d_1}{\lambda} \approx \frac{1}{2} \rightarrow h = \frac{\lambda}{d_1}\, e^{j2\pi d_1/\lambda}\left(1 + e^{j\pi}\right) = 0$$

Destructive Interference!

Multipath Channel

$h(t)$ is channel impulse response.

$$y(t) = \sum_k h(\tau_k)\, x(t - \tau_k) \;= h(t) * x(t)$$

Channel Fading: Symbols arriving along different paths sum up destructively

Example 2 paths with distance $d_1 = 1m, d_2 = 1.06m$:

$$h = \; h_1 + h_2 \; = \frac{\lambda}{d_1}\, e^{j2\pi d_1/\lambda} \; + \frac{\lambda}{d_2}\, e^{j2\pi d_2/\lambda}$$

$@f_1 = 2.5GHz\ (\lambda = 12\ cm)$: $h = 0.12\, e^{j\frac{2\pi}{3}} + 0.113\, e^{j\frac{5\pi}{3}} \approx 0.006$

$@f_2 = 5GHz\ (\lambda = 6\ cm)$: $h = 0.06\, e^{j\frac{5\pi}{3}} + 0.05\ e^{j\frac{5\pi}{3}} \approx 0.116$

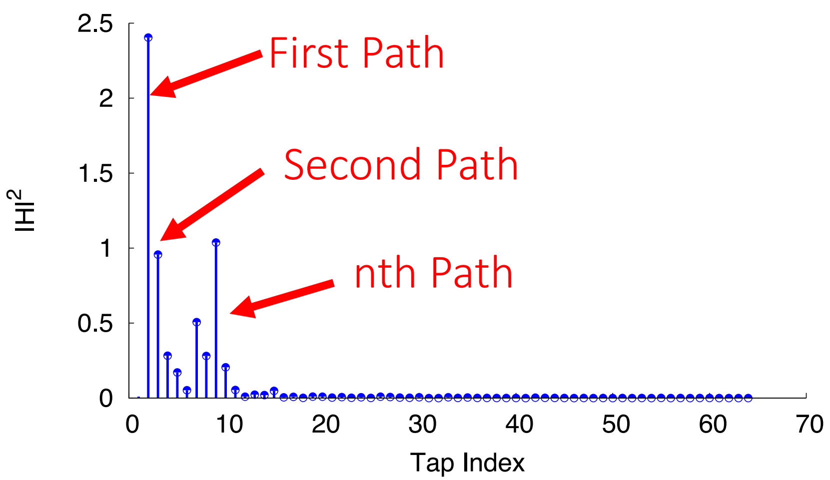
17× (24dB)

Frequency Selective Fading

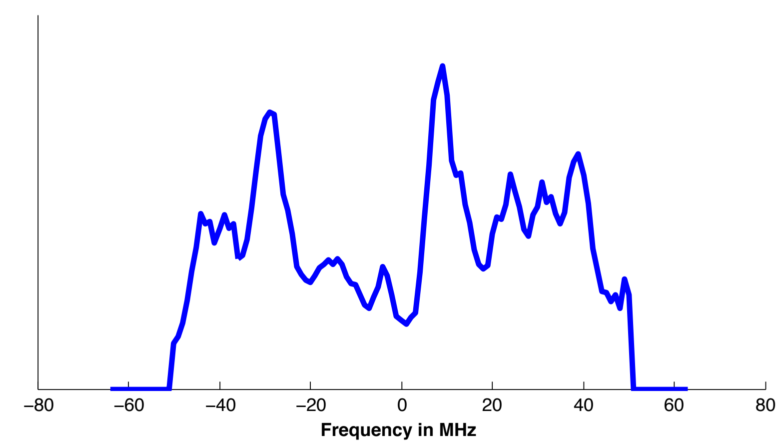
Multipath Channel

$h(t)$ is channel impulse response.

$$y(t) = \sum_k h(\tau_k)\, x(t - \tau_k) \; = h(t) * x(t) \quad \Longleftrightarrow \quad H(f)X(f)$$



Multi-tap Channel



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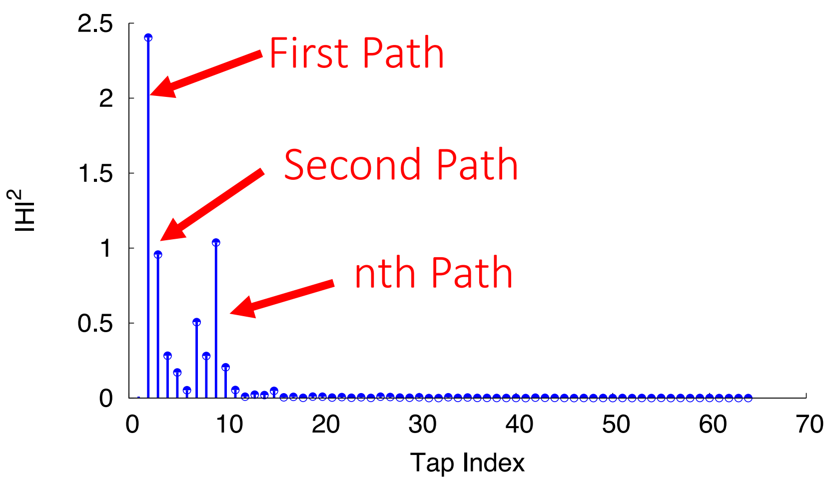
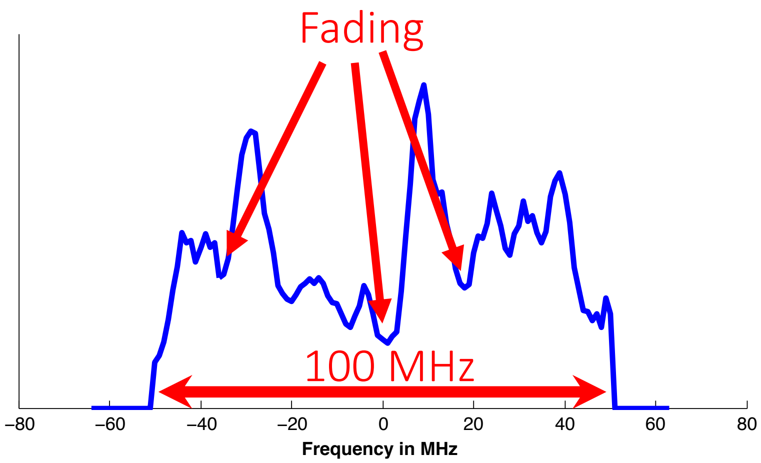




Multi-tap Channel

➡

Frequency Selective Fading

Symbols arriving along different paths sum up destructively

⬇

ISI: Inter-Symbol-Interference

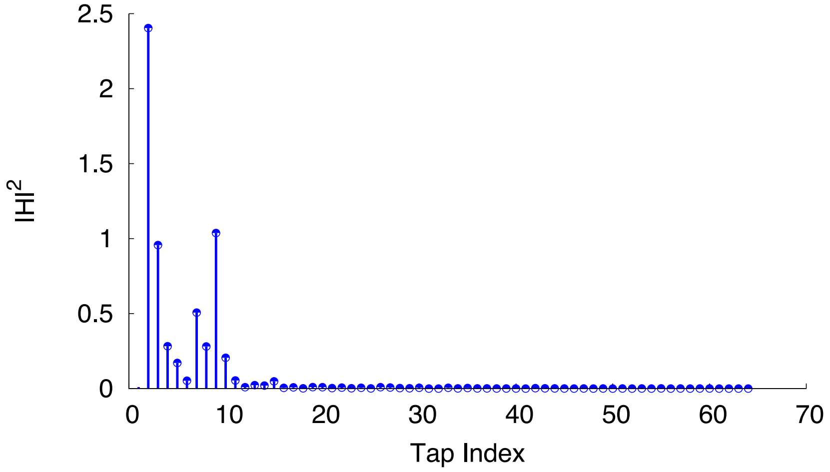
Symbols arriving along late paths interfere with following symbols.

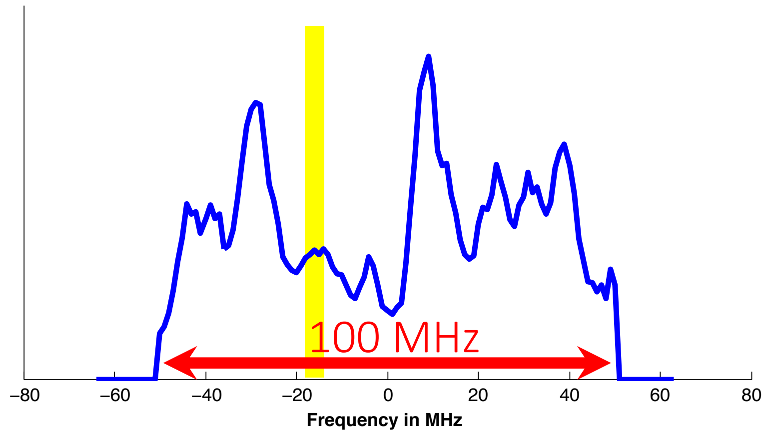
Problematic in Wideband Channel!

Narrowband Channel

$h(t)$ is channel impulse response.

$$y(t) = \sum_k h(\tau_k)\, x(t - \tau_k) \;= h(t) * x(t) \qquad \Longleftrightarrow \qquad H(f)X(f)$$





Narrowband Channel

$h(t)$ is channel impulse response.

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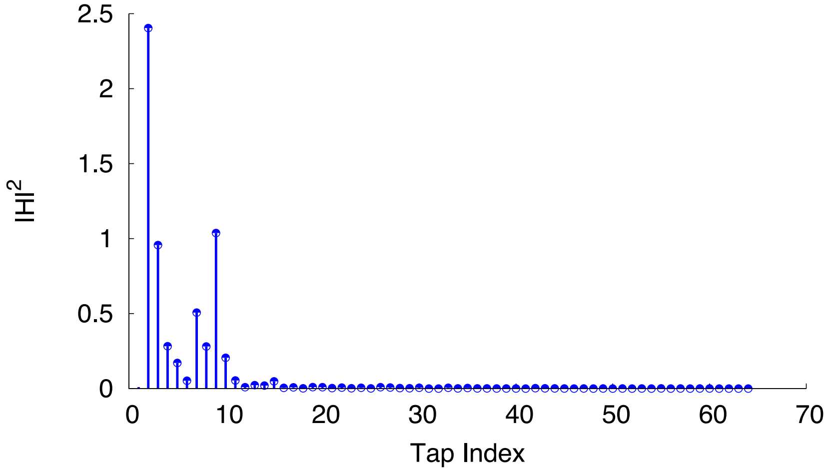

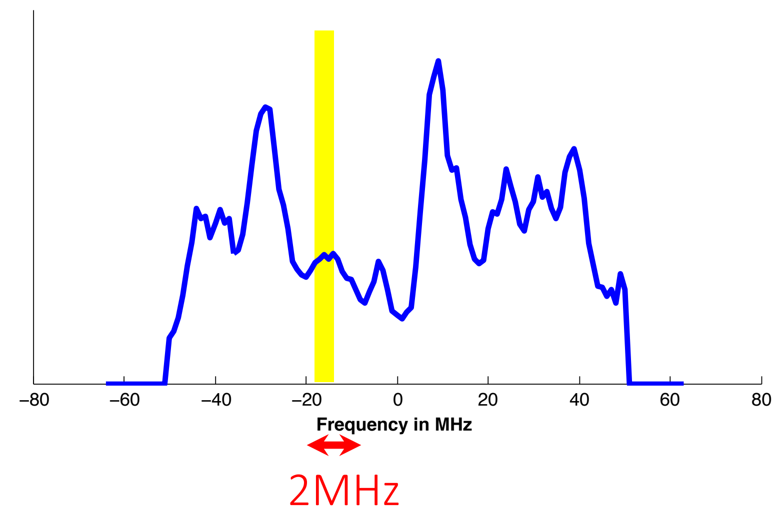

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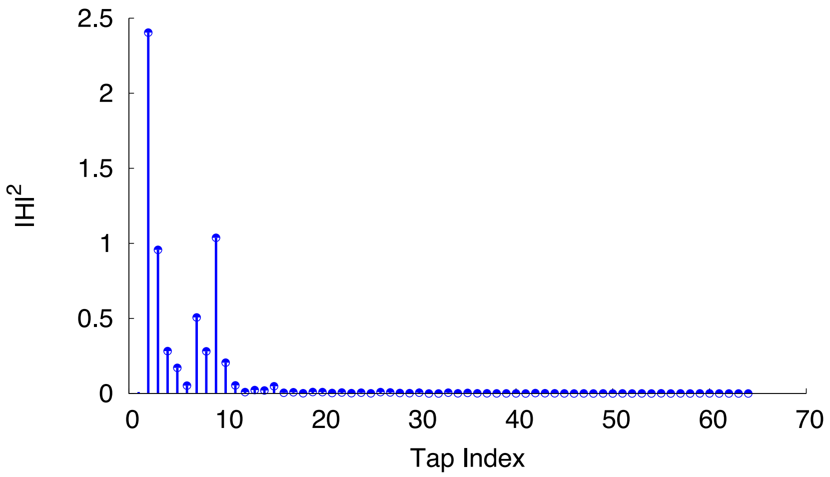

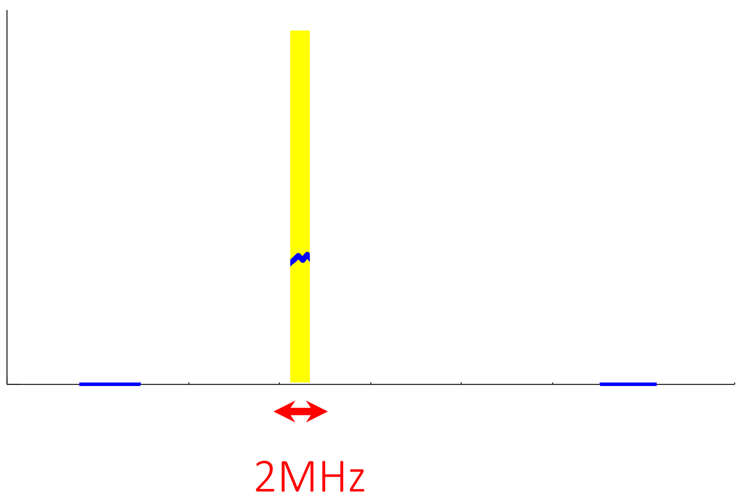

Narrowband Channel

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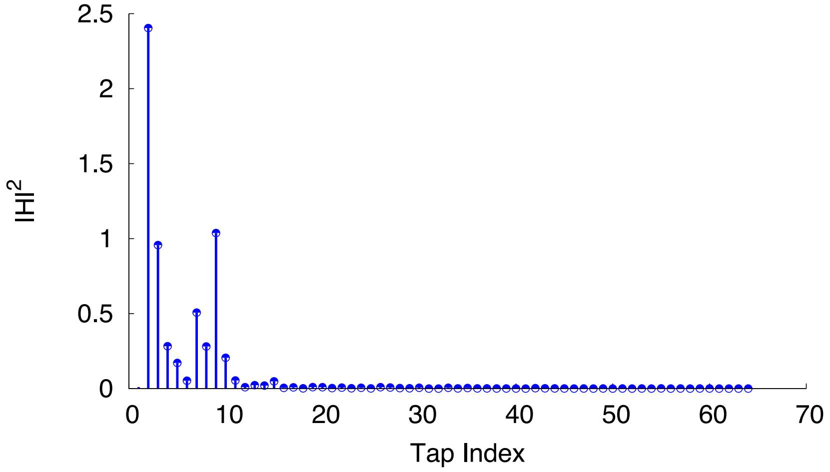
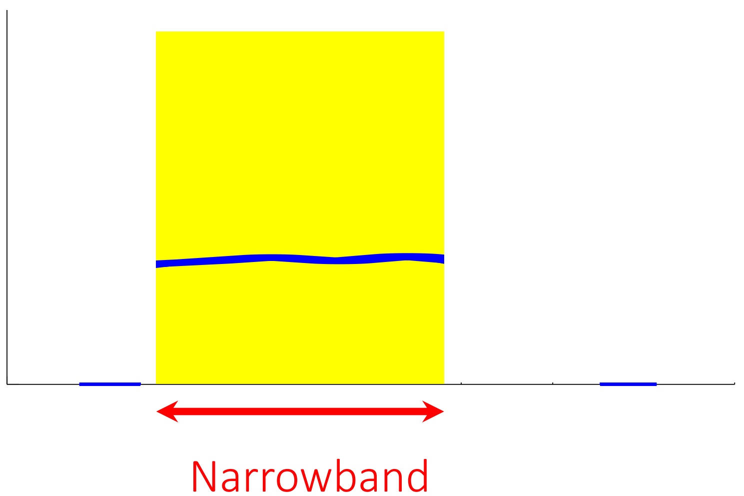




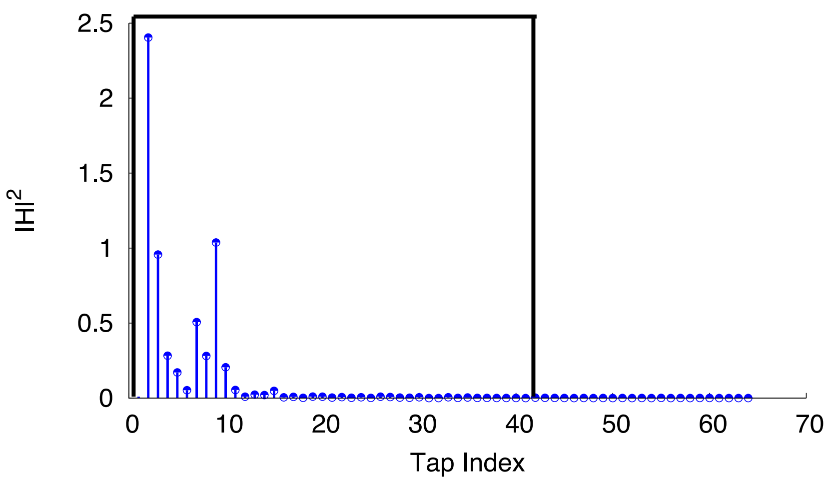
Symbol time: $T \propto \frac{1}{\text{Bandwidth}}$ $\gg \tau_k$

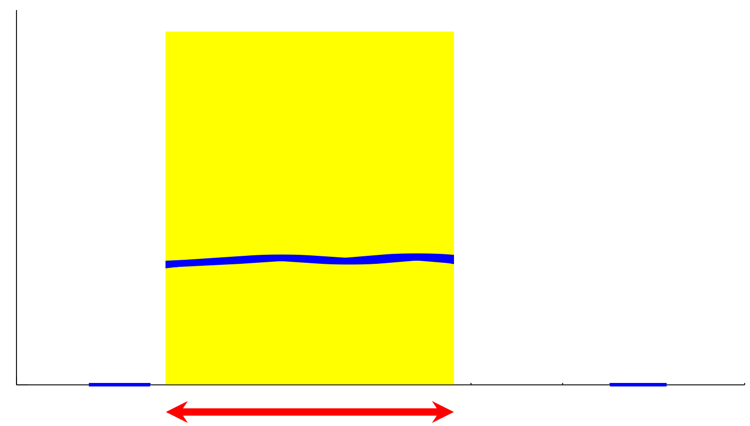
Flat Channel

Narrowband Channel

$h(t)$ is channel impulse response.

$$y(t) = \sum_k h(\tau_k)\, x(t - \tau_k) \;= h(t) * x(t) \qquad \Longleftrightarrow \qquad H(f)X(f)$$





Narrowband

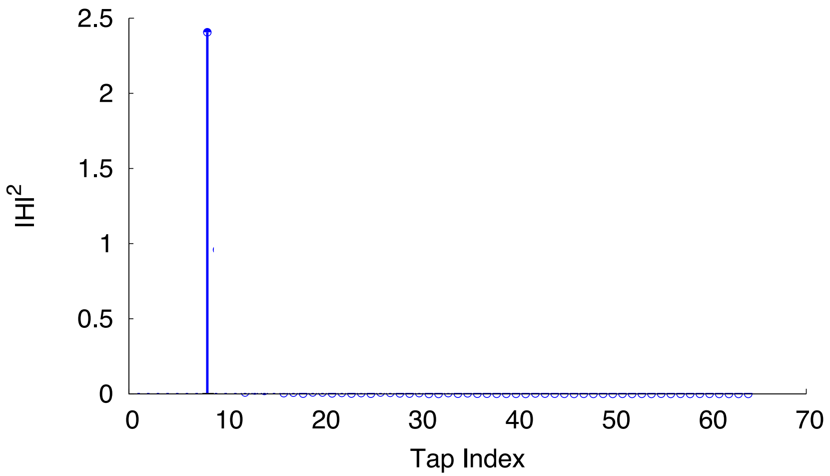
Symbol time: $T \propto \frac{1}{\text{Bandwidth}} \gg \tau_k$

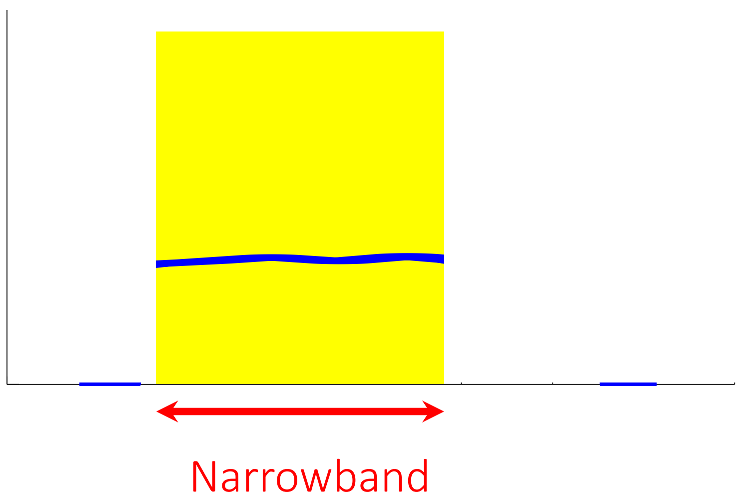
Flat Channel

Narrowband Channel

$h(t)$ is channel impulse response.

$$y(t) = \sum_k h(\tau_k)\, x(t-\tau_k) \;= h(t) * x(t) \qquad \Leftrightarrow \qquad H(f)X(f)$$





Symbol time: $T \propto \frac{1}{\text{Bandwidth}}$ $\gg \tau_k$

Flat Channel

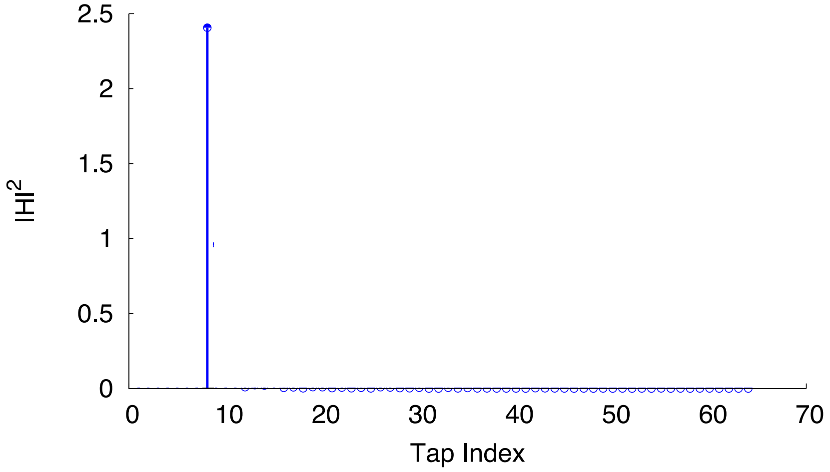
$$y(t) = \sum_k h(\tau_k)\, x(t-\tau_k) \;\approx \sum_k h(\tau_k)\, x(t) \;= \left(\sum_k h(\tau_k)\right) x(t) \;= hx(t)$$

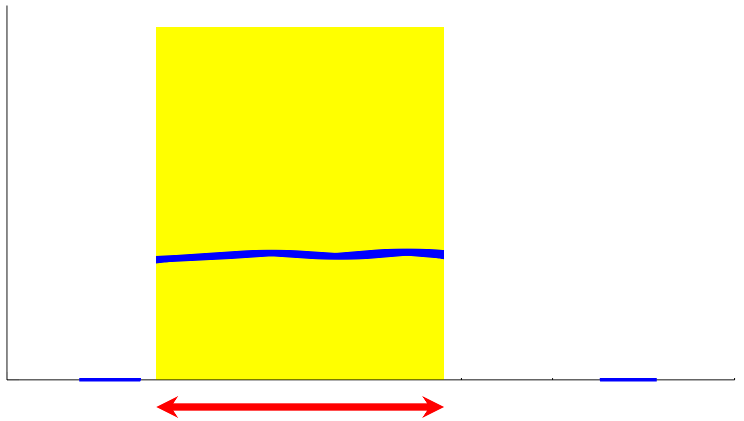
Narrowband Channel is Approximated by a Single Tap Channel

Narrowband Channel

$h(t)$ is channel impulse response.

$$y(t) = \sum_k h(\tau_k)\, x(t) \; = h\, x(t) \qquad \Leftrightarrow \qquad h\, X(f)$$





Symbol time: $T \propto \frac{1}{\text{Bandwidth}} \gg \tau_k$

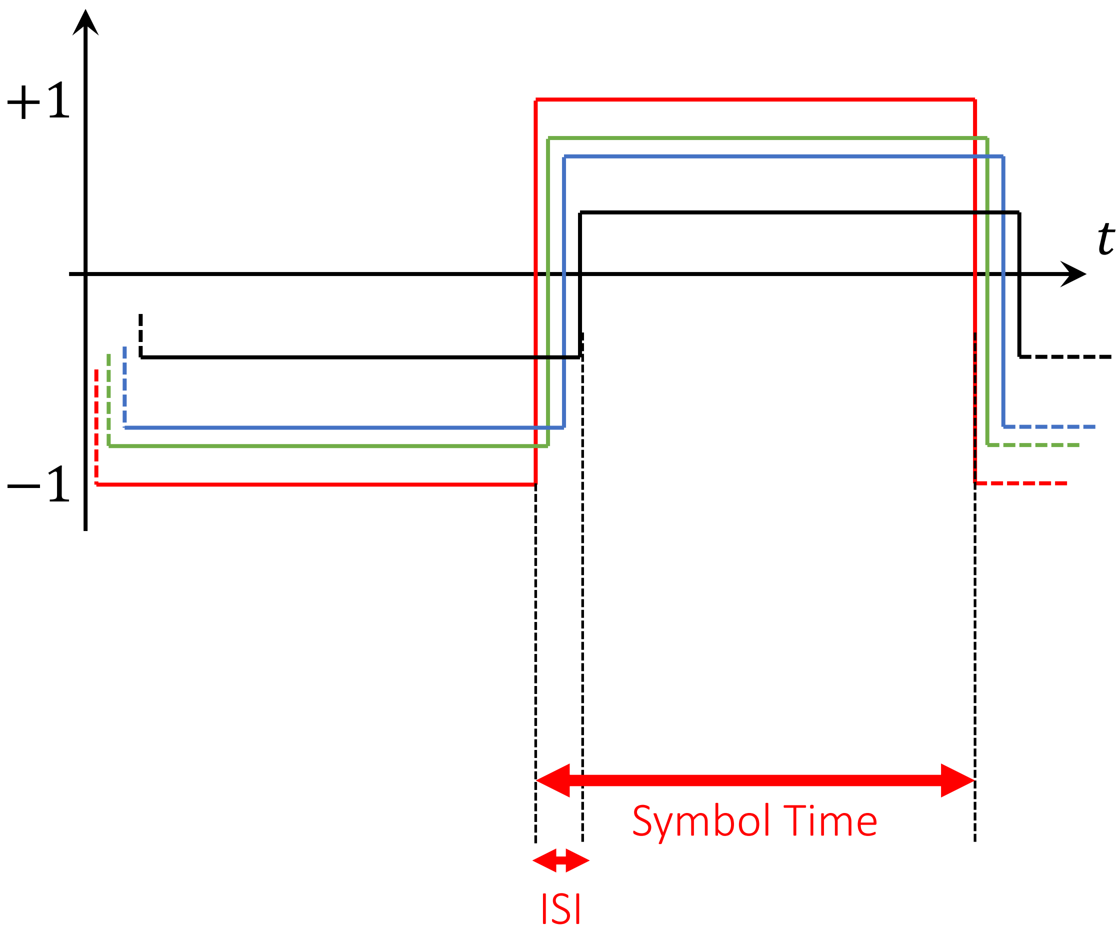
$$y(t) = \sum_k h(\tau_k)\, x(t-\tau_k) \;\approx \sum_k h(\tau_k)\, x(t) \; = \left(\sum_k h(\tau_k)\right) x(t) \; = hx(t)$$

Narrowband Channel is Approximated by a Single Tap Channel

Narrowband vs. Wideband Channel

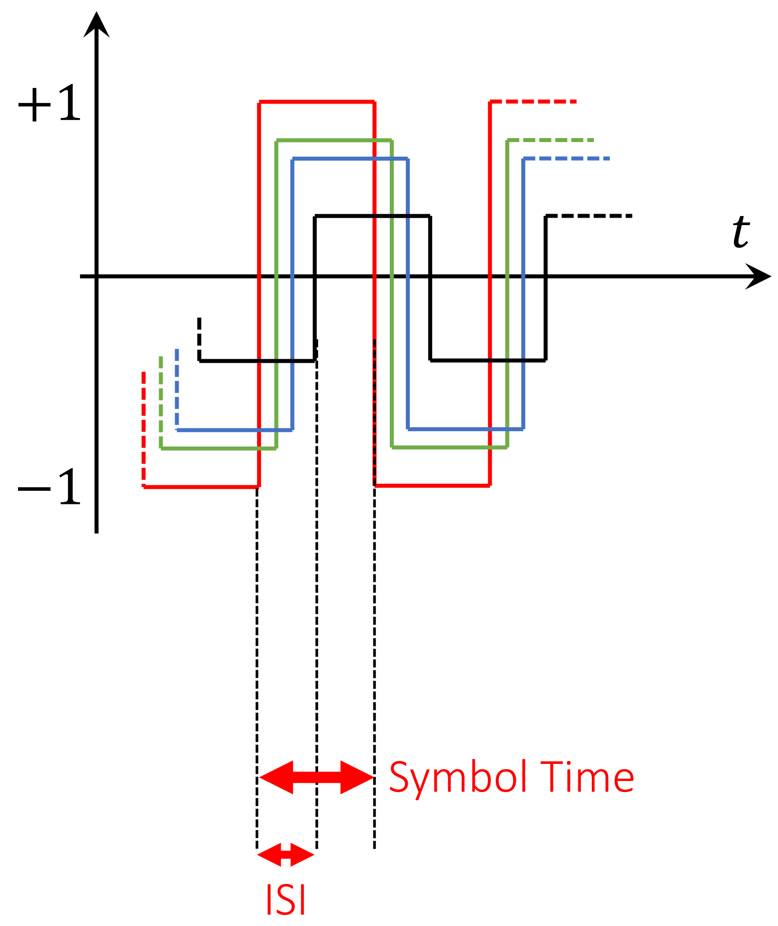
Narrowband Channel

$$h\,x(t)$$



Wideband Channel

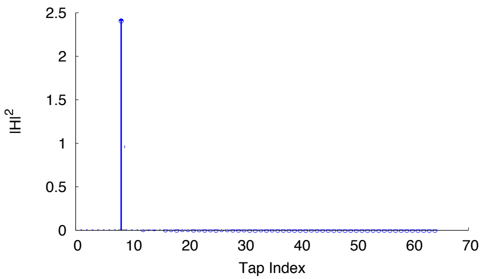
$$h(t) * x(t)$$



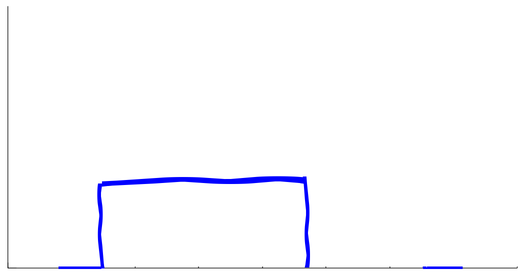
Narrowband vs. Wideband Channel

Narrowband Channel

$h\,x(t) \quad \Leftrightarrow \quad h\,X(f)$



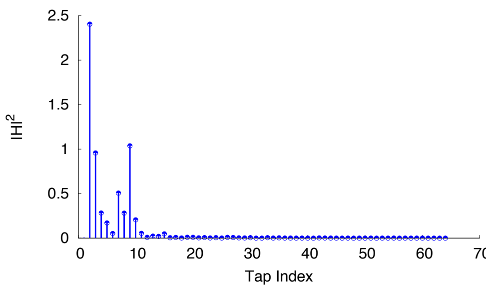
≈ Single Tap



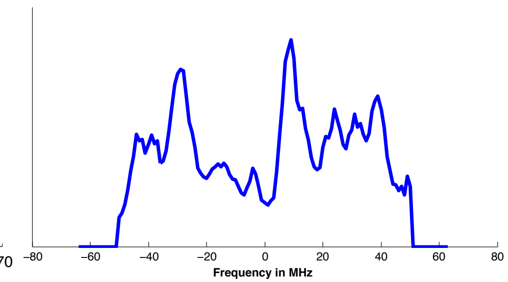
≈ Flat Channel

Wideband Channel

$h(t) * x(t) \quad \Leftrightarrow \quad H(f)\,X(f)$



Multi-tap Channel (ISI)



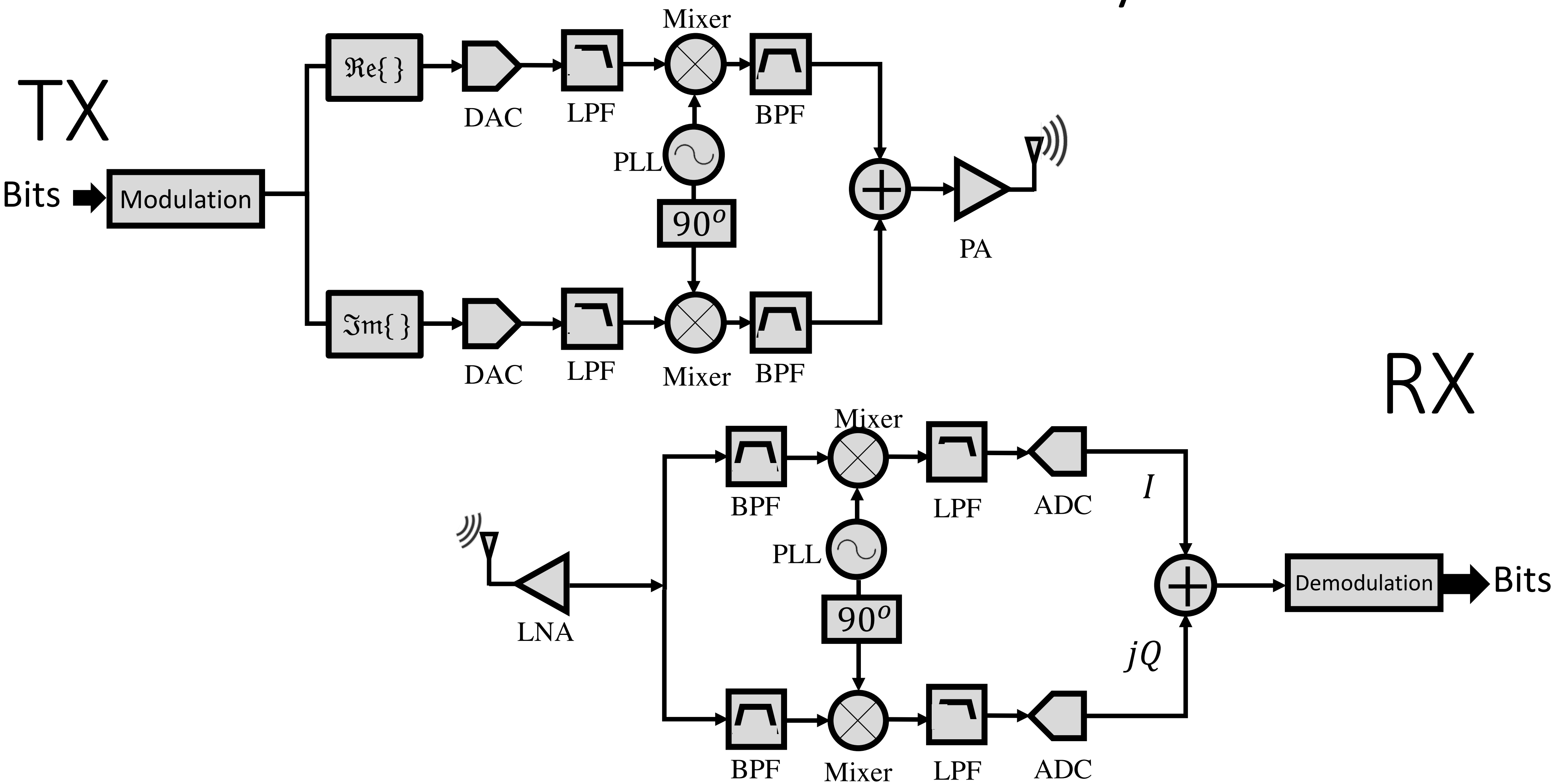
Frequency Selective Channel

Need to correct for ISI to be able to decode correctly!

Estimating and Correcting for multi-tap channel is hard

Simplify processing using: OFDM

Wireless Communication System



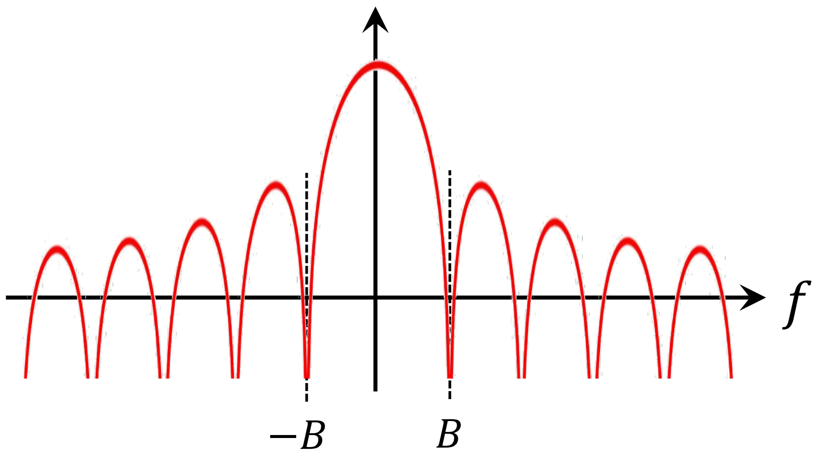
Single Carrier Modulation

Symbols modulated on a single carrier frequency: $s[n]e^{-j2\pi f_c t}$

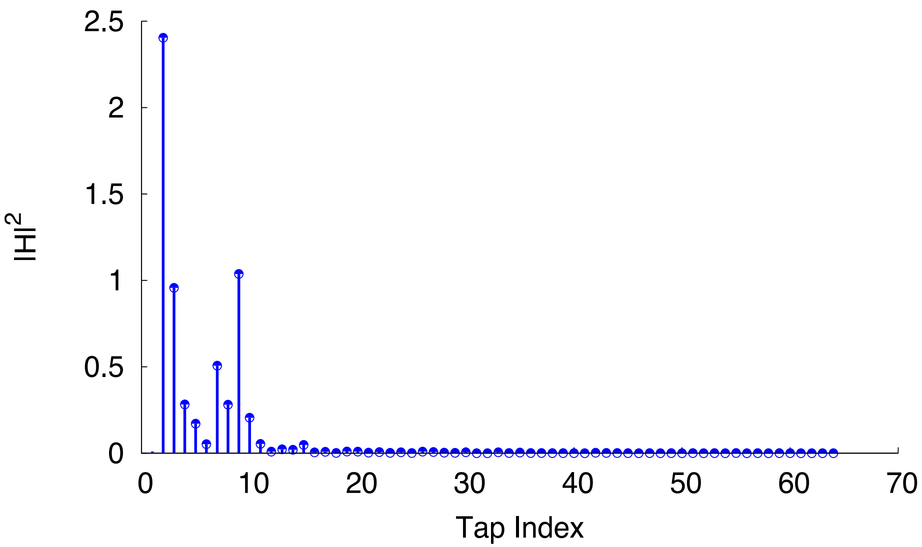
Single Carrier Modulation

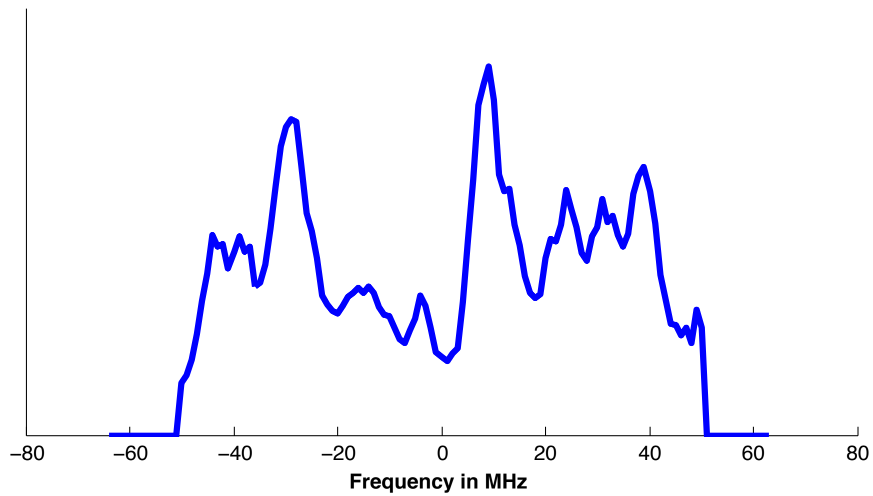
Symbols modulated on a single carrier frequency

- Low Spectral Efficiency: sinc & raised cosine leakage



- ISI: Inter-Symbol-Interference limits performance

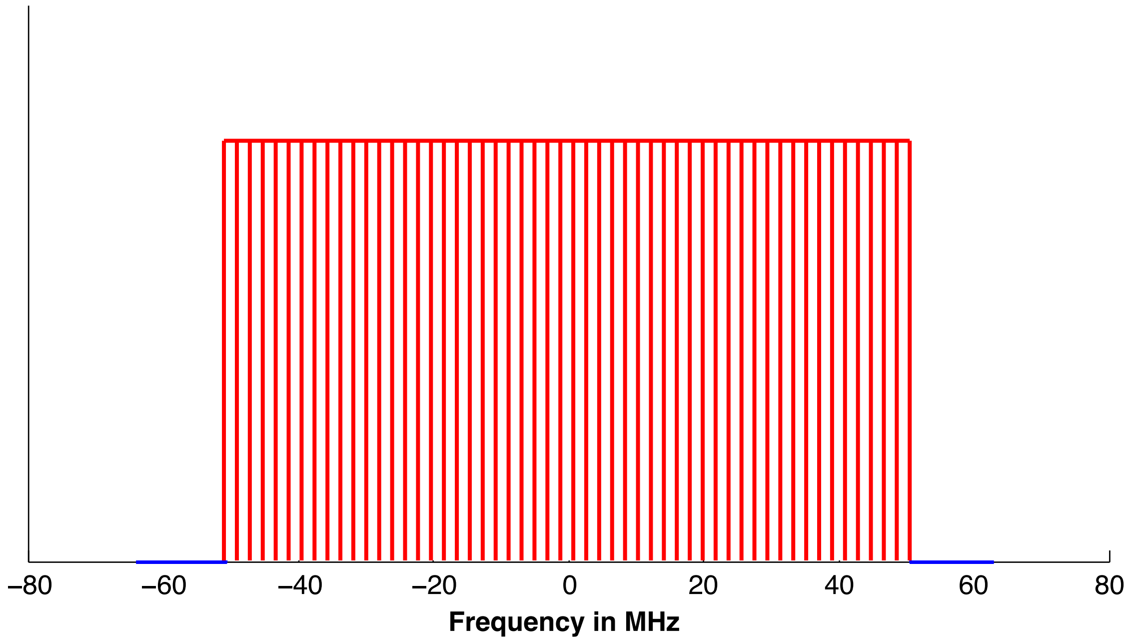




Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

- Divide spectrum into many narrow bands



$$x(t) = \sum_i s_i[n]\mathrm{e}^{-j2\pi f_i t}$$

- Transmit symbols on different carriers in narrow bands

- Channel is Flat → No need to worry about ISI

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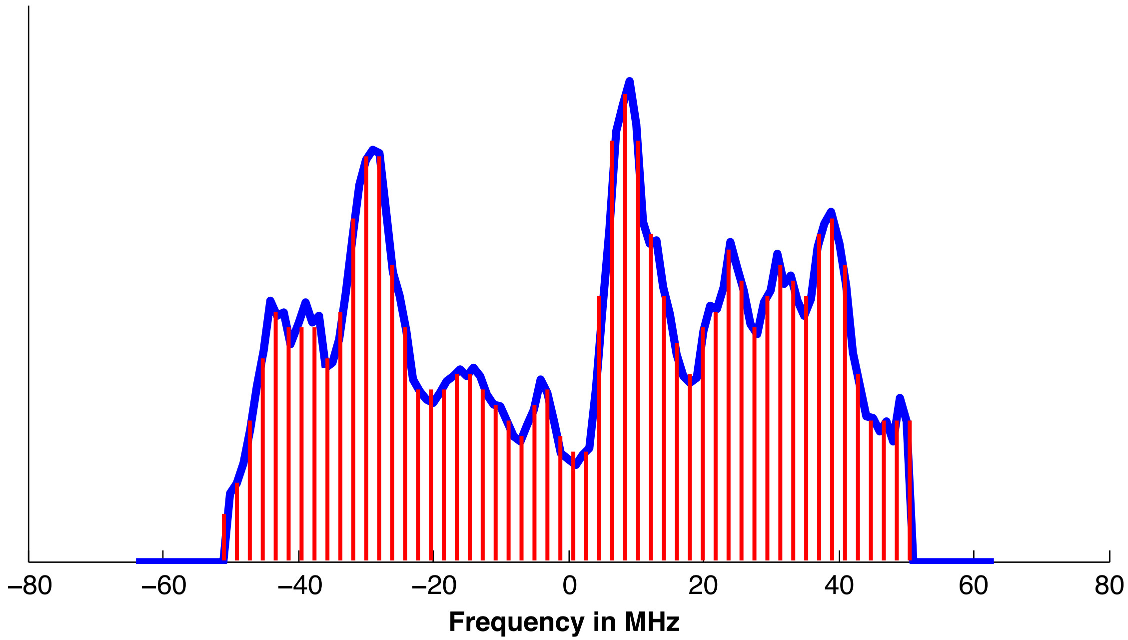

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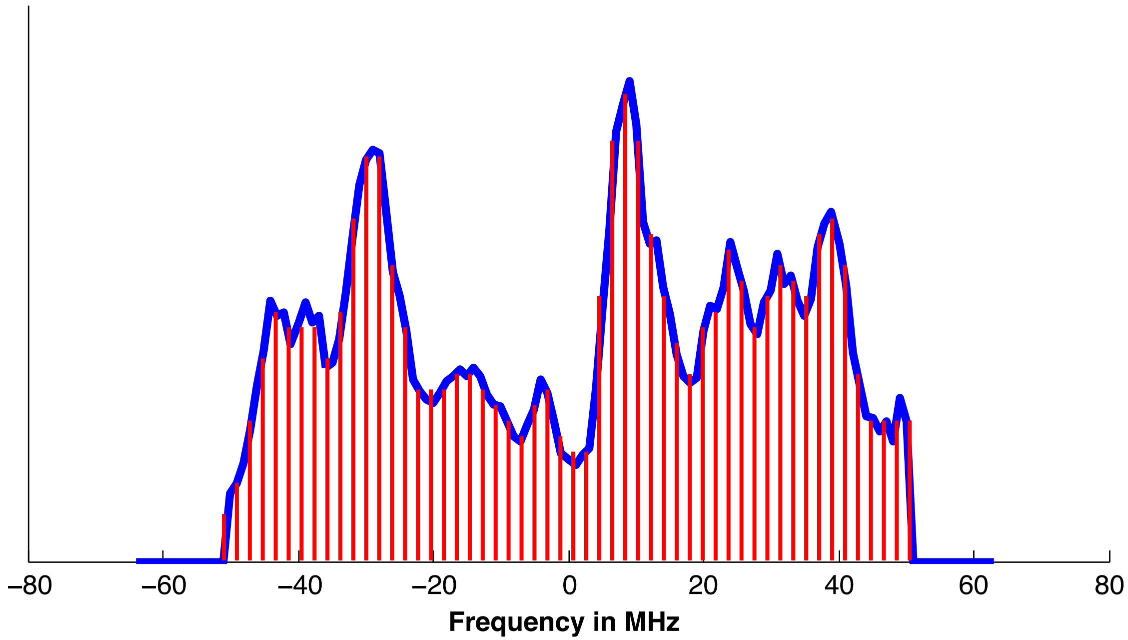

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Not That Simple!

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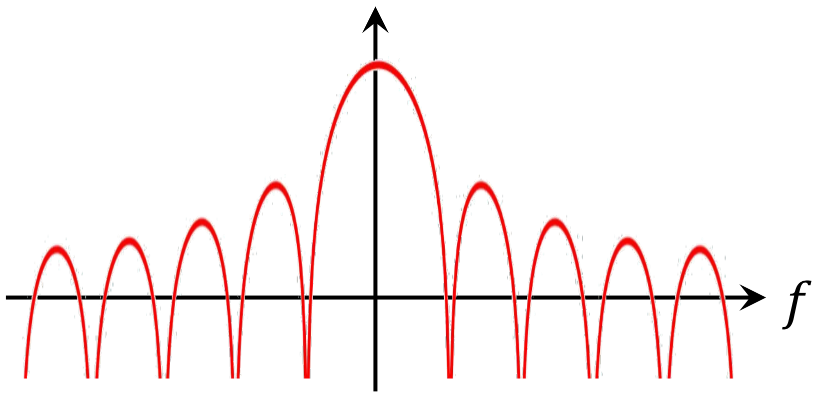
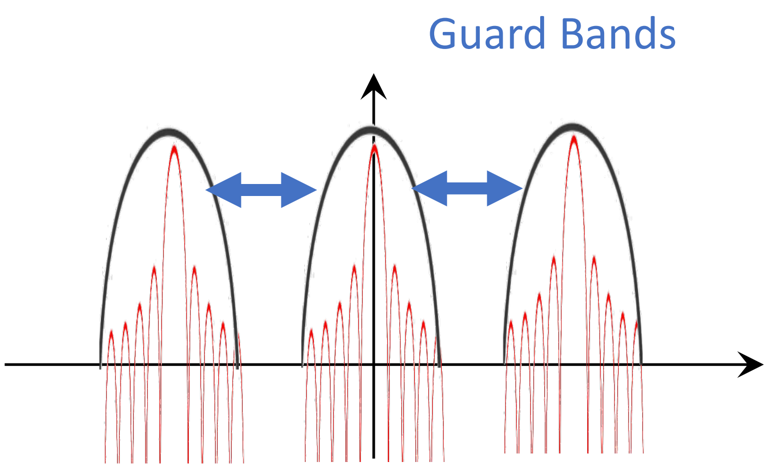




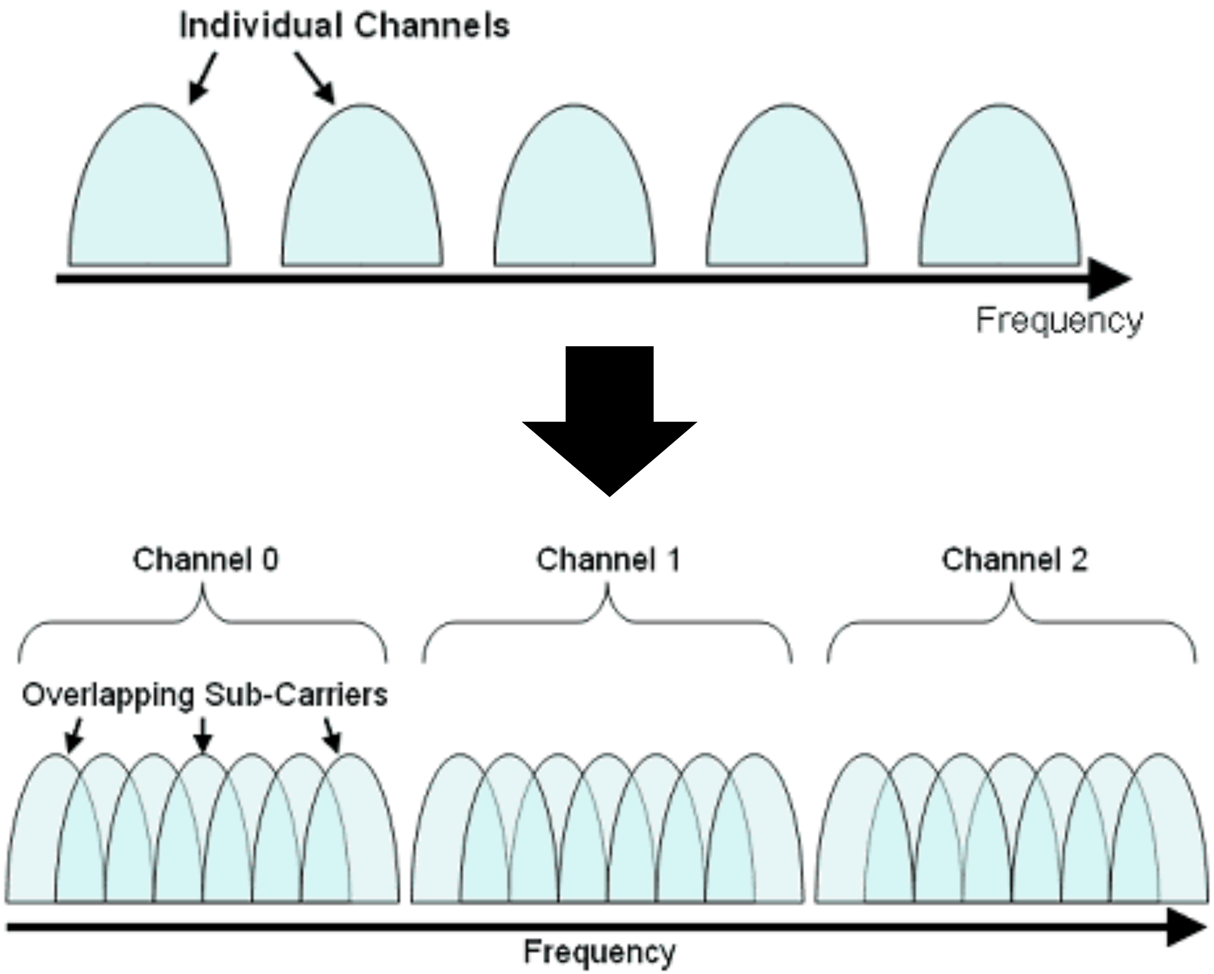
- Significant Leakage between adjacent subcarriers
- Need Guard Bands → Very inefficient!

Solution: Make the Sub-Carriers Orthogonal

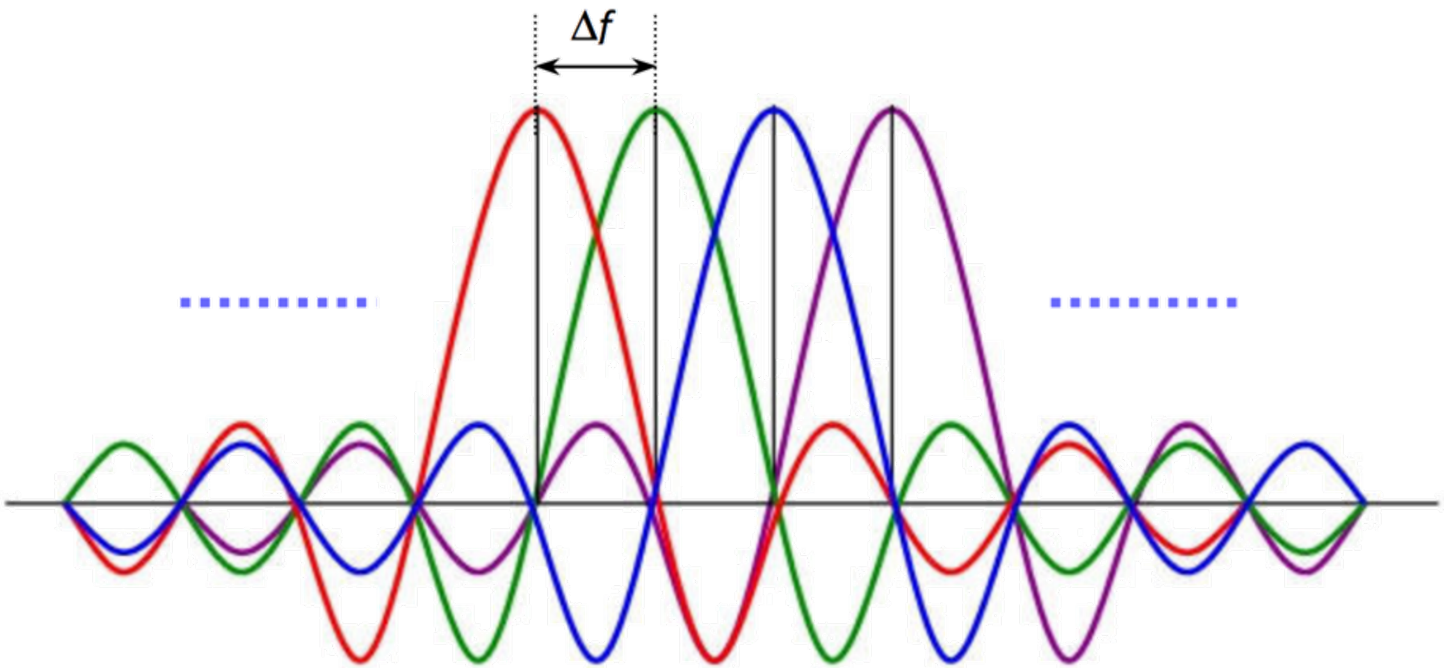
Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

Make the Sub-Carriers Orthogonal



OFDM: Orthogonal Frequency Division Multiplexing



- Subcarriers are orthogonal: At the sub-carrier frequency, the sampled value has zero leakage from other subcarriers.

- Subcarrier separation can be very small, for N subcarriers and bandwidth B:

$$\Delta f = \frac{B}{N}$$

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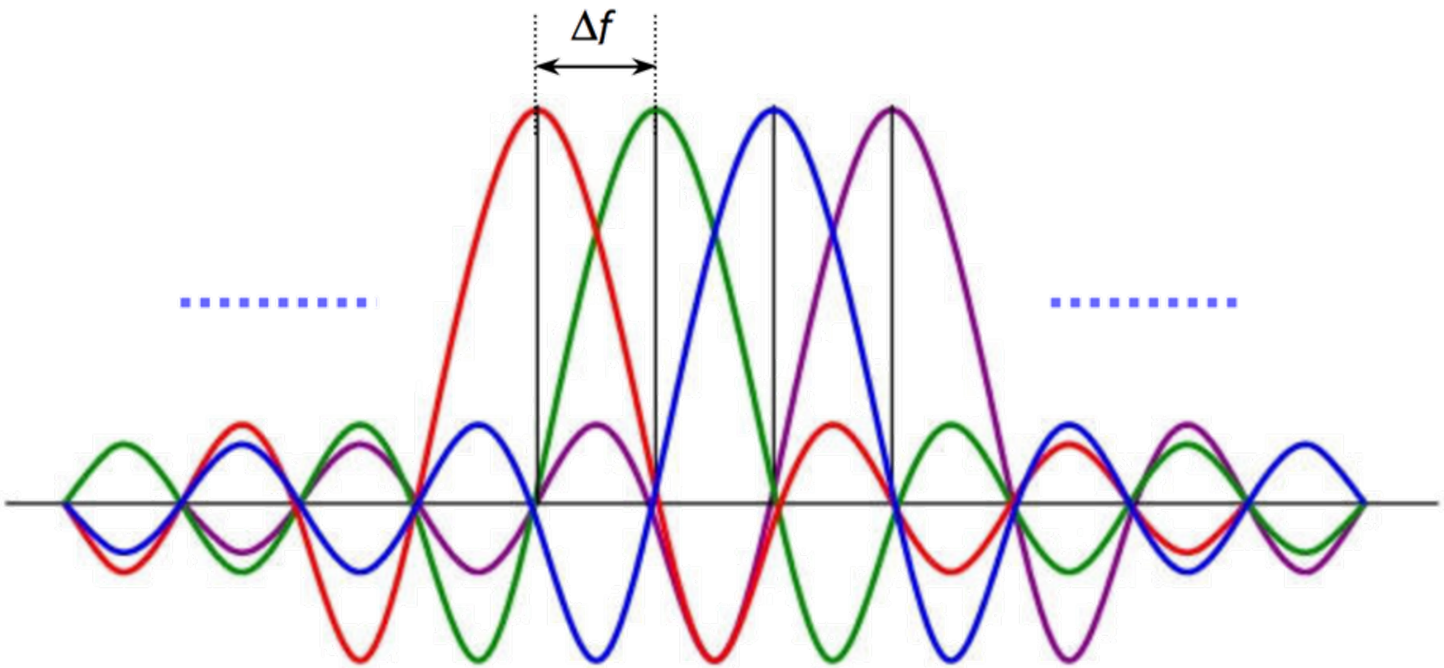

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How to Achieve This?

OFDM: Orthogonal Frequency Division Multiplexing

Use DFT: Discrete Fourier Transform

N-Point DFT: $X(f_i) = \frac{1}{N}\sum_{t=0}^{N-1} x(t)e^{-j\frac{2\pi f_i t}{N}}$

N-Point IDFT: $x(t) = \sum_{f_i=0}^{N-1} X(f_i)e^{j\frac{2\pi f_i t}{N}}$

Send symbols in Frequency Domain

$X(f_i) = s[n] \rightarrow$ Compute and transmit $x(t)$ using IDFT

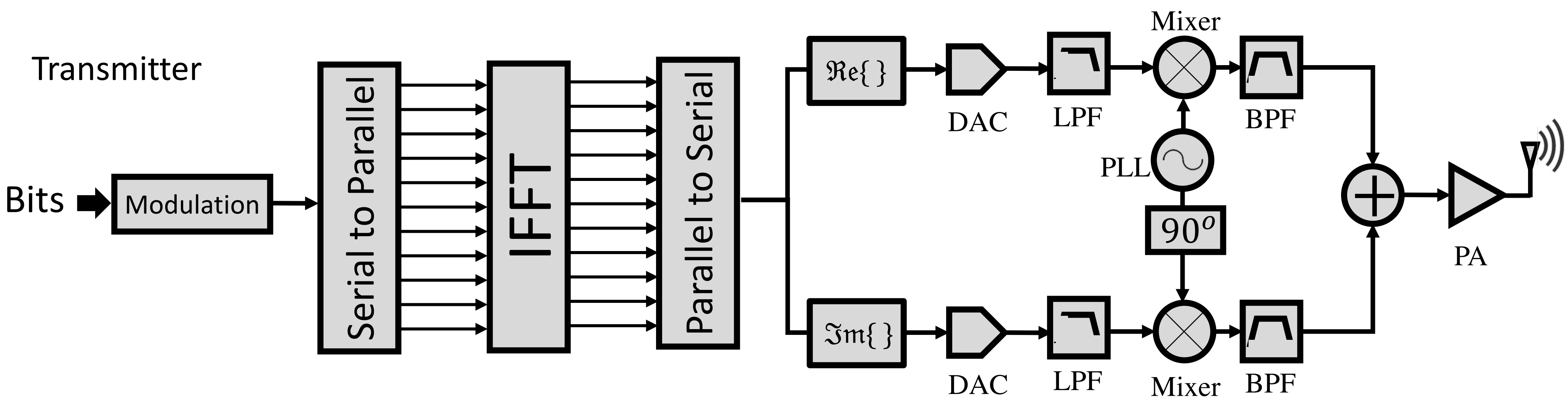
OFDM: Orthogonal Frequency Division Multiplexing

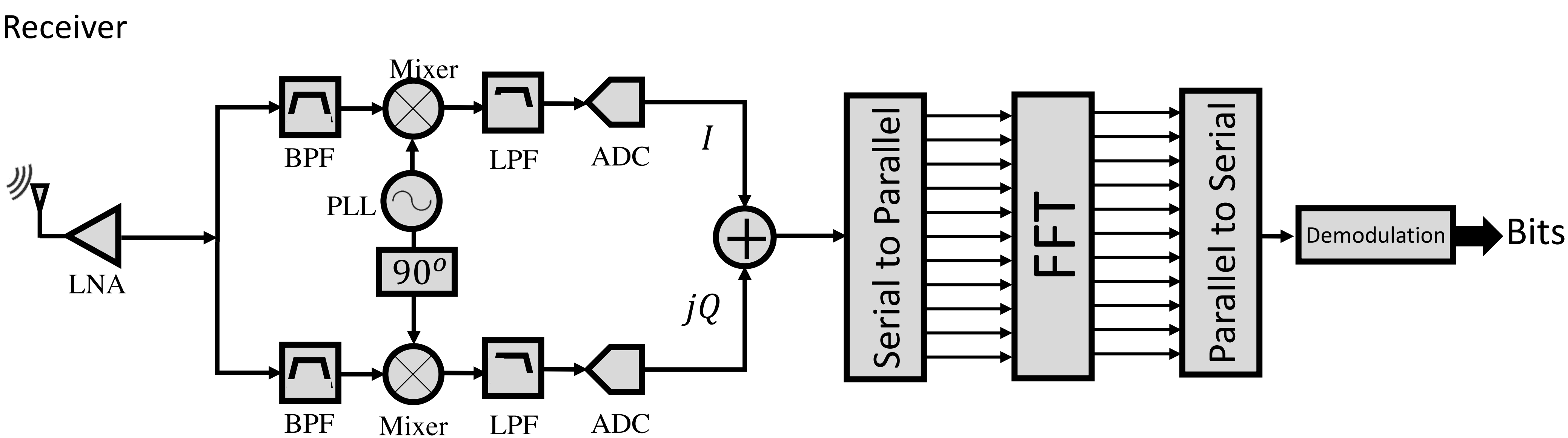
Send symbols in Frequency Domain

$X(f_i) = s[n] \rightarrow$ Compute and transmit $x(t)$ using IDFT

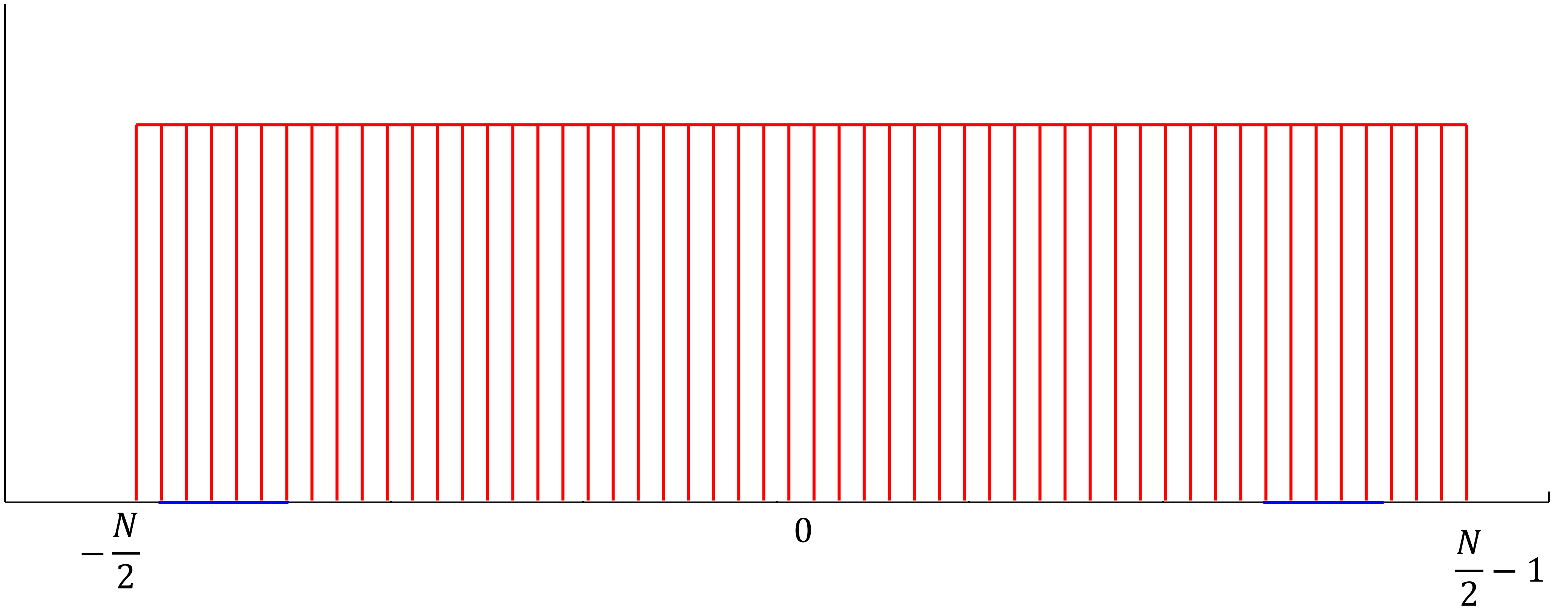
- N subcarrier → IDFT of length N
- Symbols $s[n]$ can come from any modulation: BPSK, QPSK, QAM…
- $x(t)$ is complex → need I & Q → No point using PAM or ASK …
- OFDM Symbol: N samples of $x(t)$ generated from the same modulated symbols using IDFT.
- OFDM Symbol Time: $T = N/B$ where B is the bandwidth.
- OFDM Frequency Bin Width: $\Delta f = 1/T = B/N$

OFDM: Orthogonal Frequency Division Multiplexing





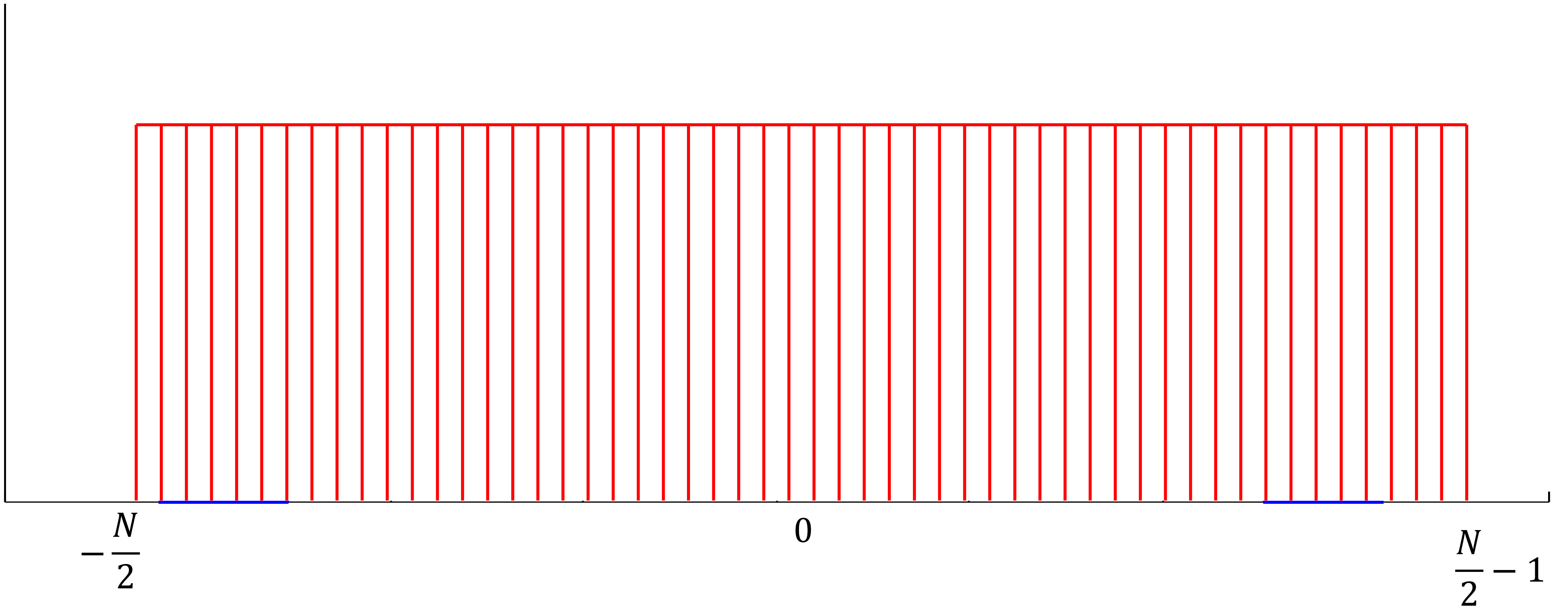
OFDM Symbol in Frequency Domain



- FFT can be represented 0 to $N-1$ or $N/2$ to $N/2\ -1$.
- OFDM Symbol created in digital baseband → 0 bin corresponds to DC

$$X(0)=\frac{1}{N}\sum_{t=0}^{N-1}x(t)e^{-j\frac{2\pi 0t}{N}}\ =\frac{1}{N}\sum_{t=0}^{N-1}x(t)\ =DC$$

OFDM Symbol in Frequency Domain



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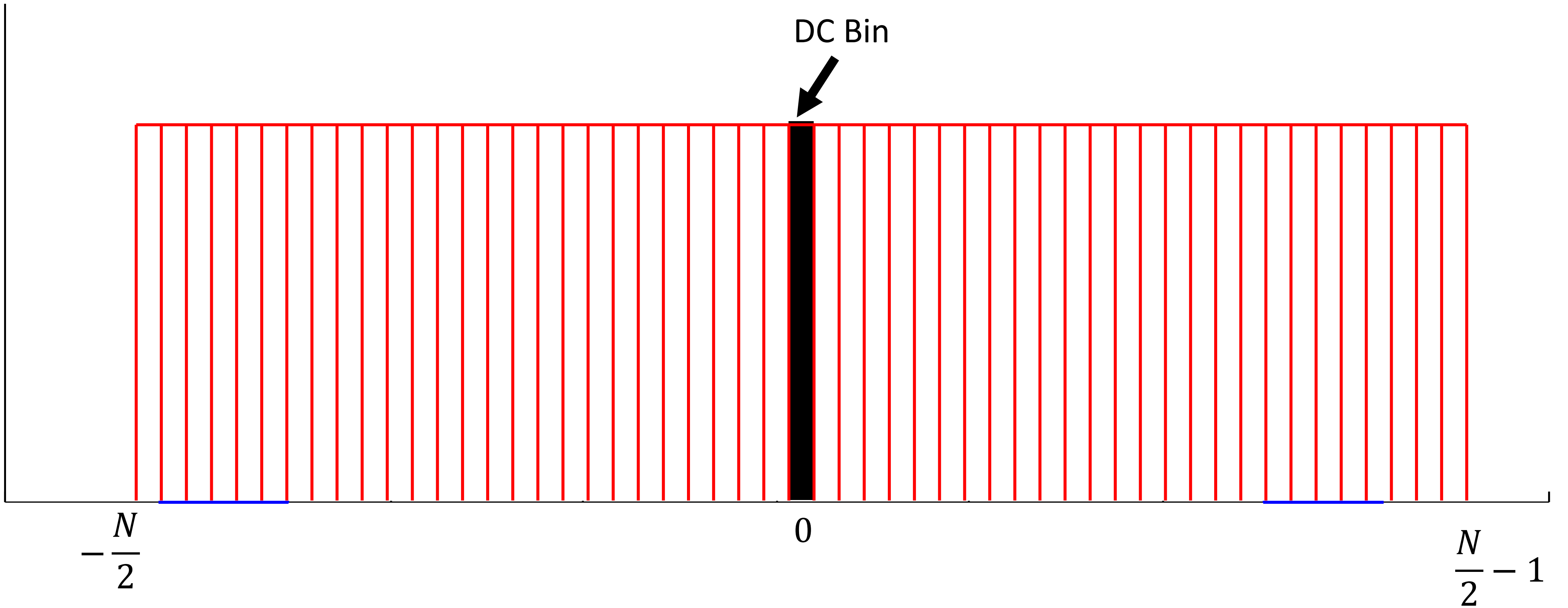

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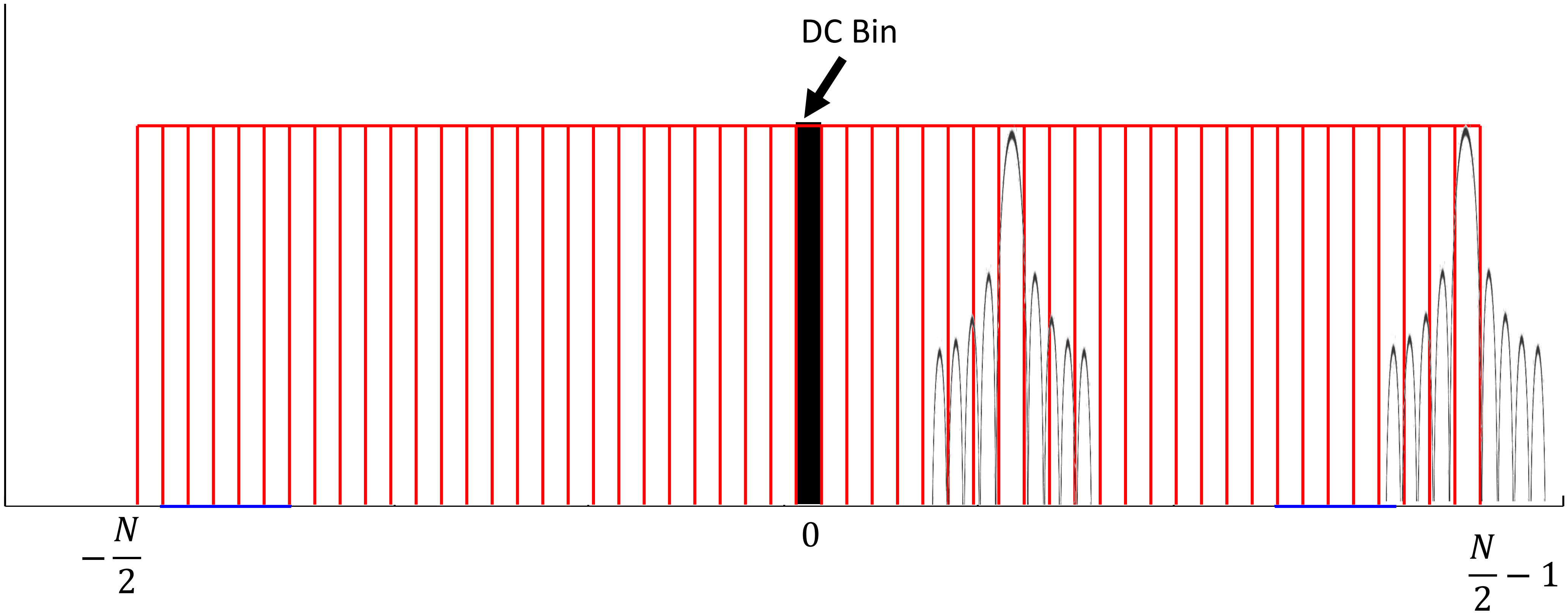
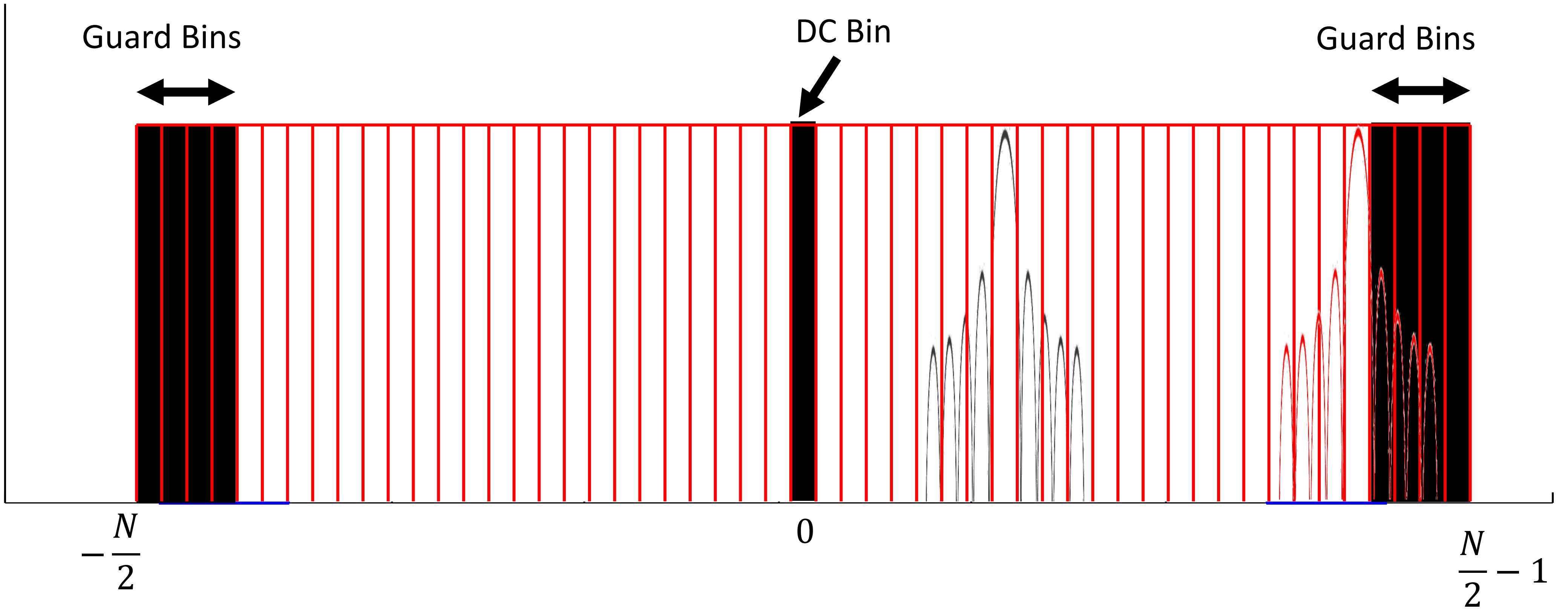


- Subcarriers orthogonal to each other but not to near by channels.
- Need Guard Bins at sides of the channel → Transmit nothing there

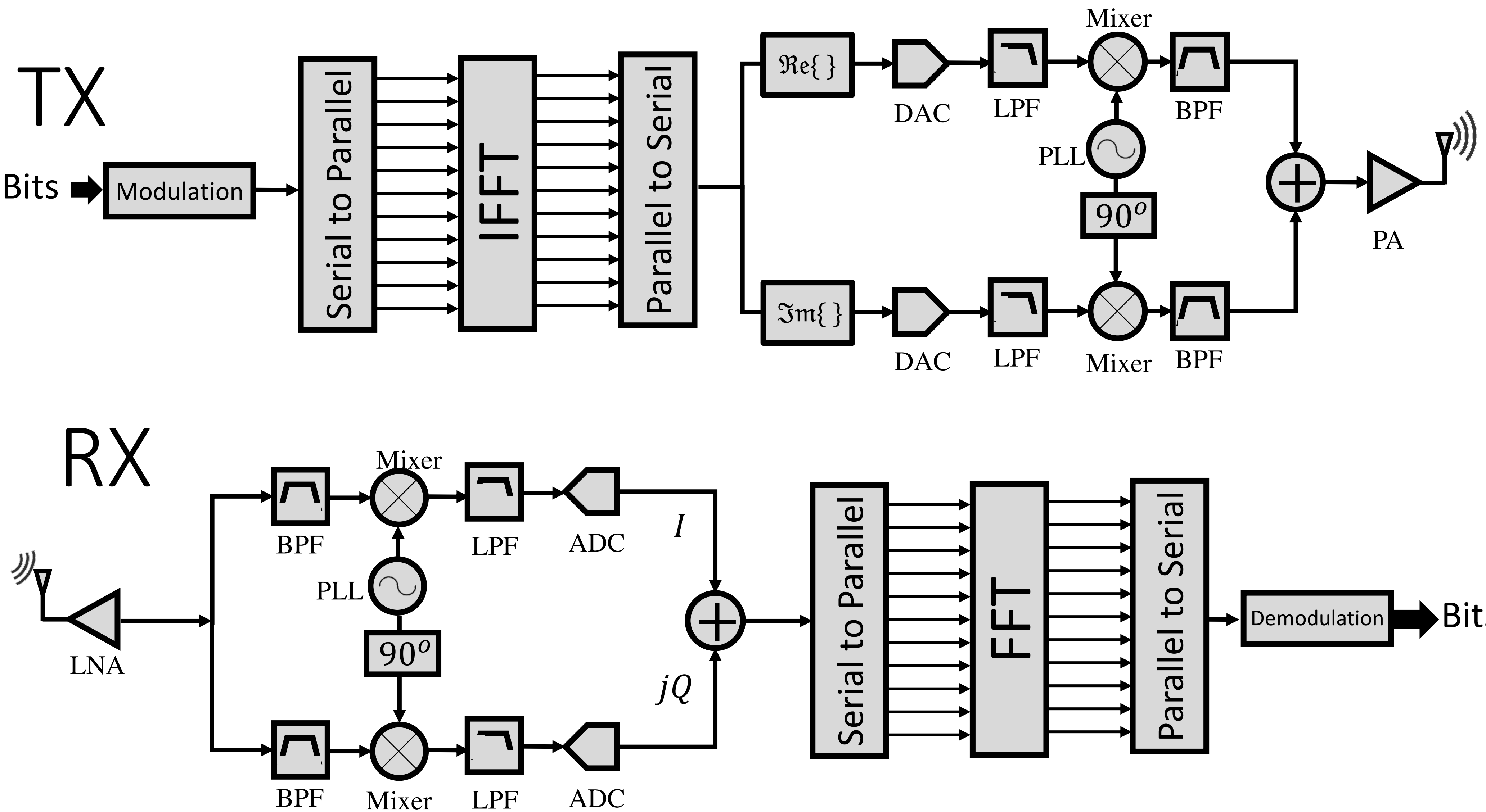
OFDM Symbol in Frequency Domain



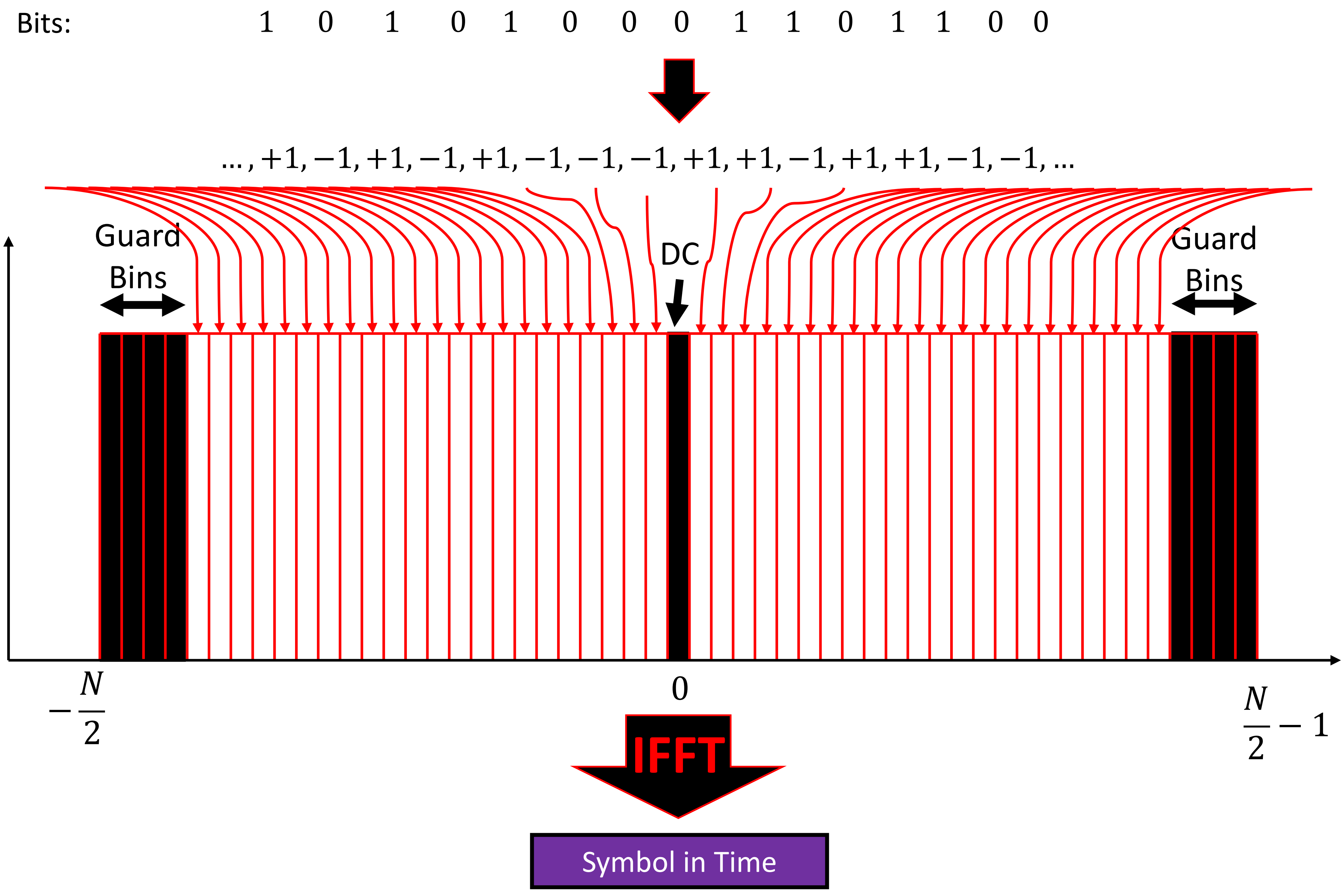
- Subcarriers orthogonal to each other but not to near by channels.
- Need Guard Bins at sides of the channel → Transmit nothing there
- Reduce Number of Guard band from N to 2 → Very Spectrally Efficient

OFDM: Orthogonal Frequency Division Multiplexing

Transmit Symbols in Frequency Domain On Orthogonal Subcarriers

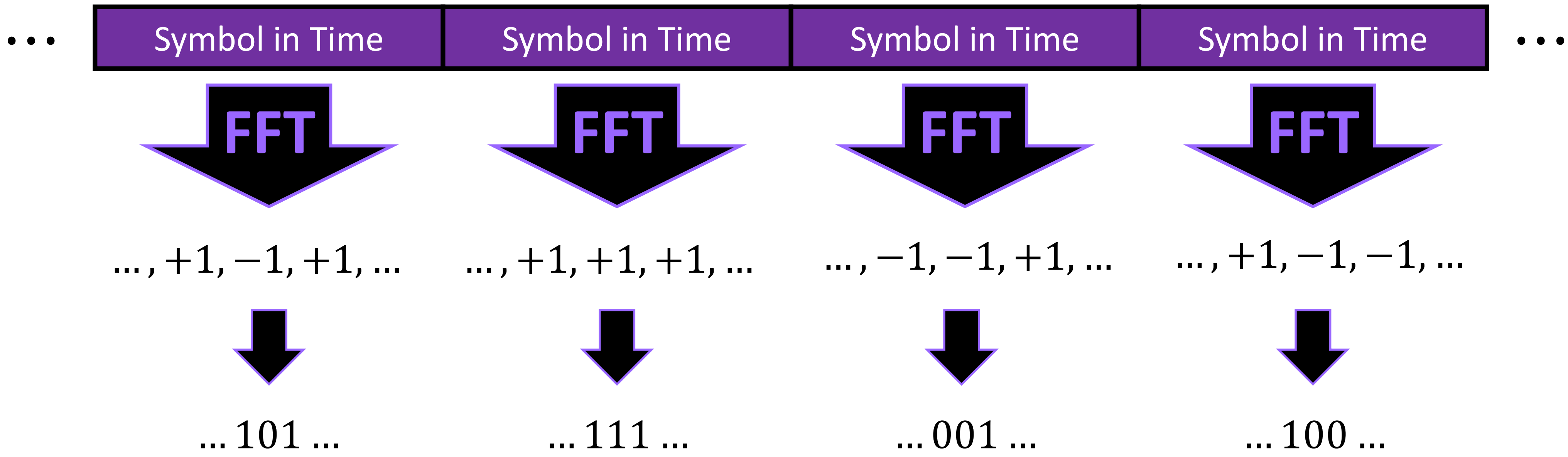


OFDM Symbol



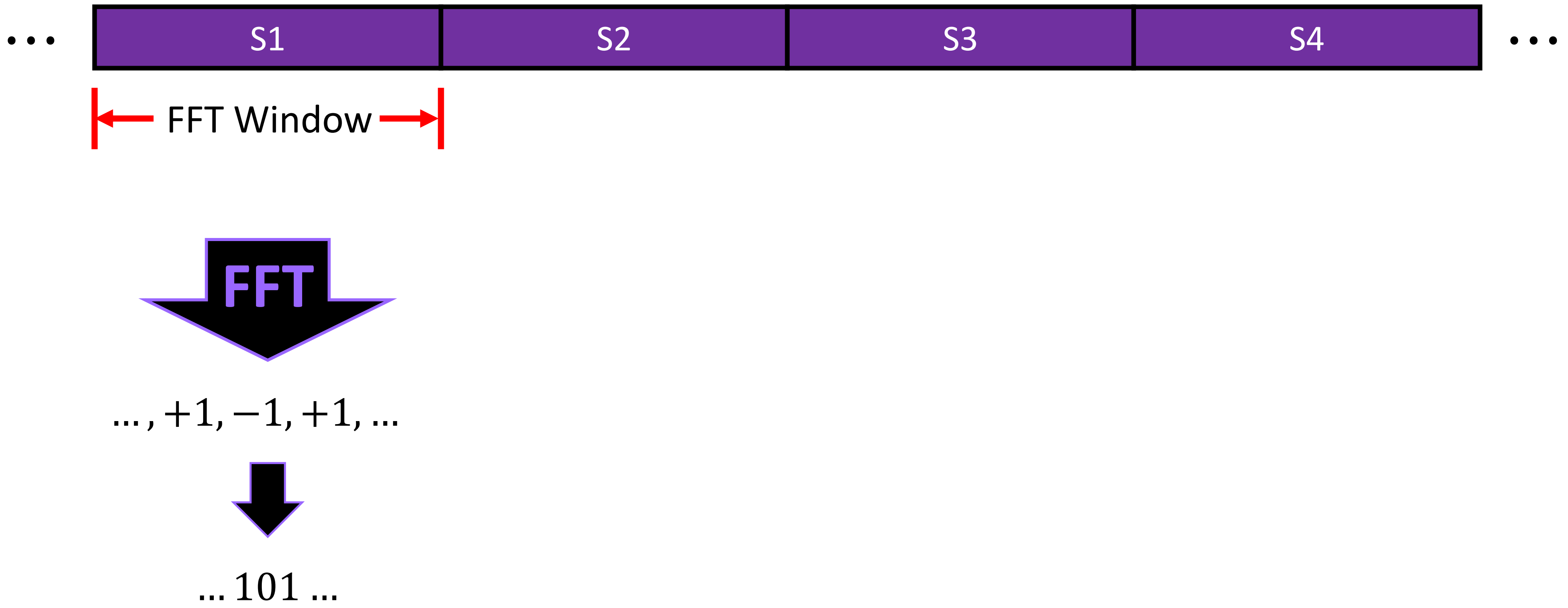
Bits:
1 0 1 0 1 0 0 0 1 1 0 1 1 0 0
…, +1, −1, +1, −1, +1, −1, −1, −1, +1, +1, −1, +1, +1, −1, −1, …
Guard Bins
DC
Guard Bins
$-\frac{N}{2}$
0
$\frac{N}{2}-1$
IFFT
Symbol in Time

OFDM Symbol

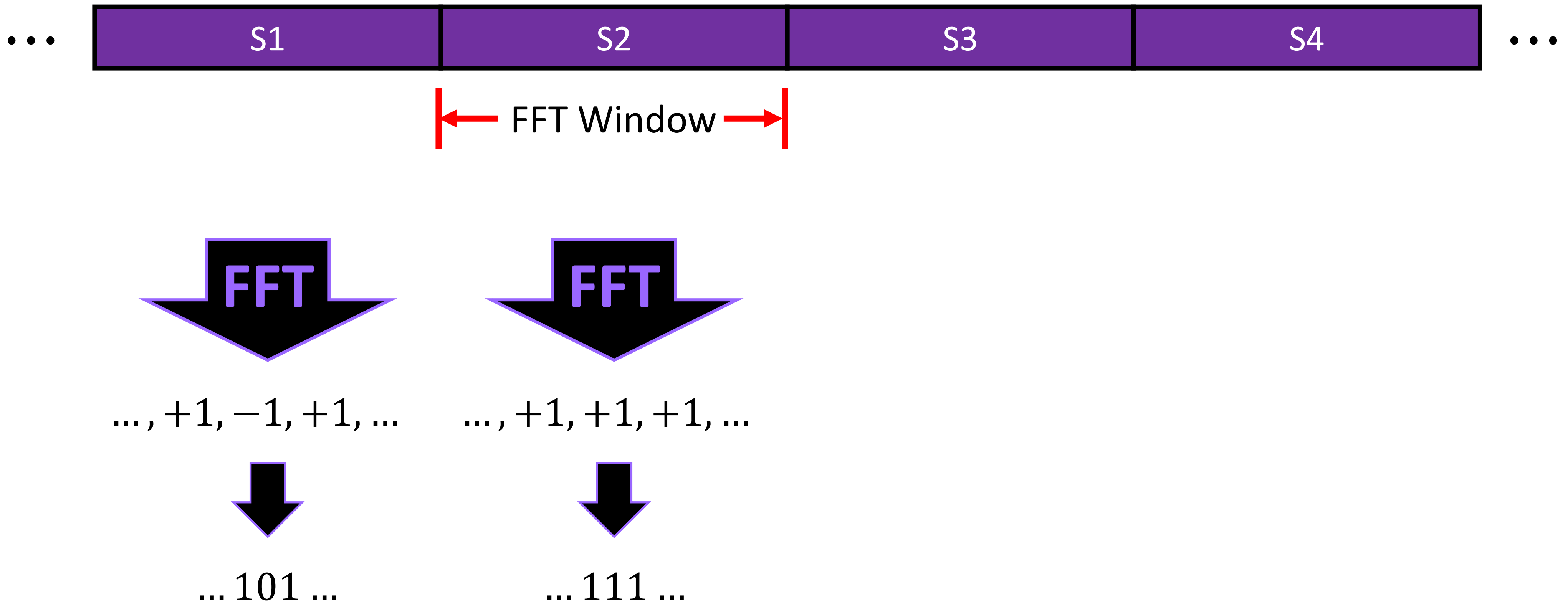


Not That Simple

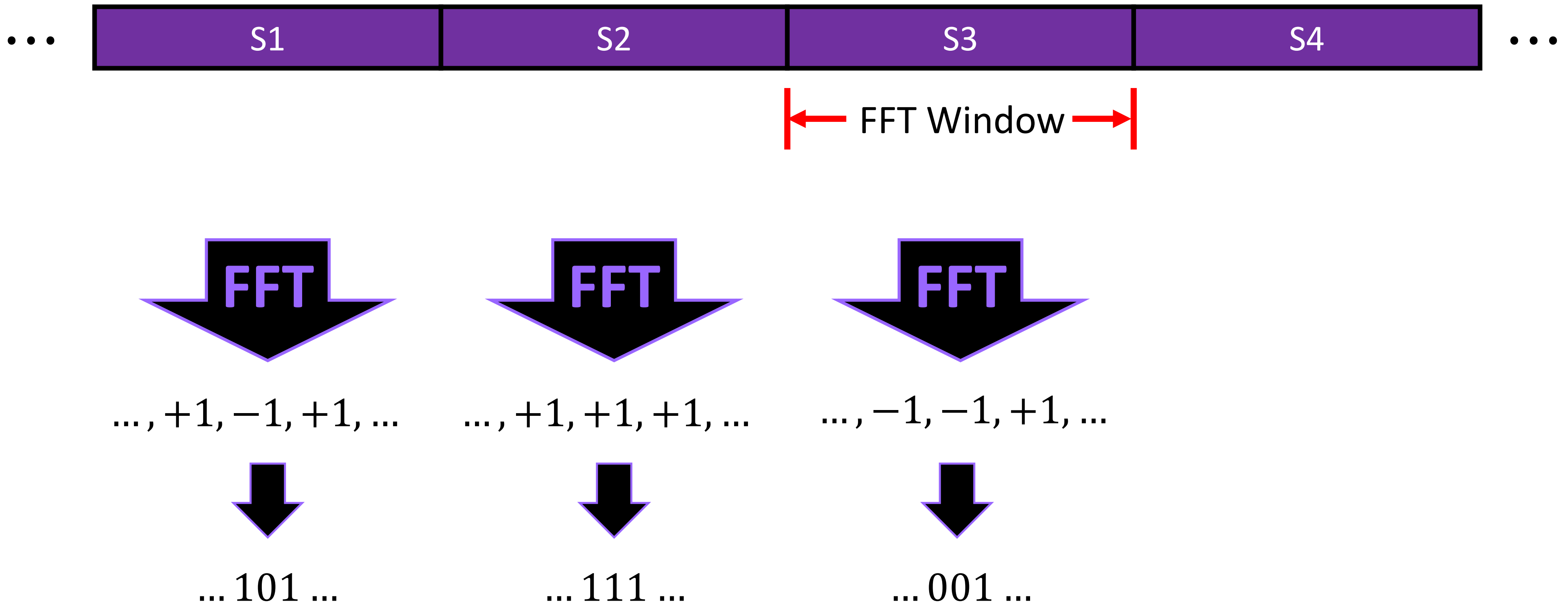
OFDM Symbol



OFDM Symbol

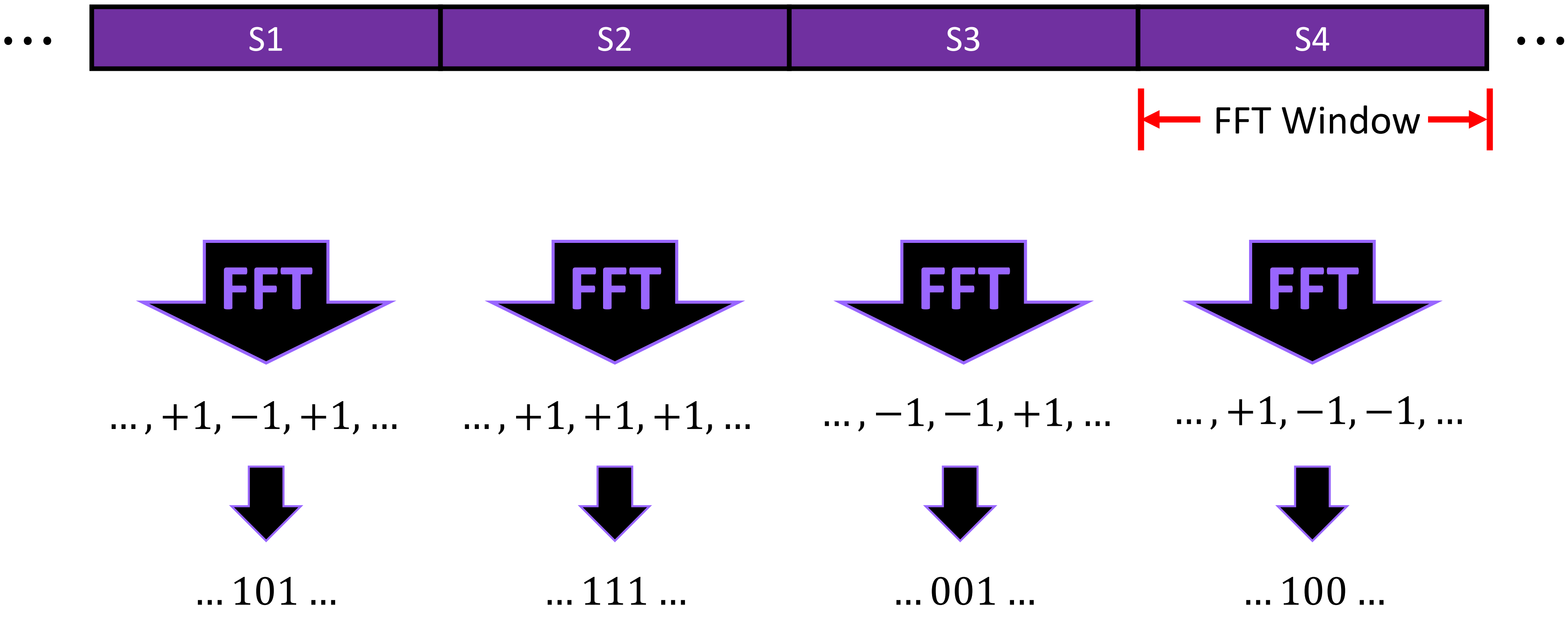


OFDM Symbol



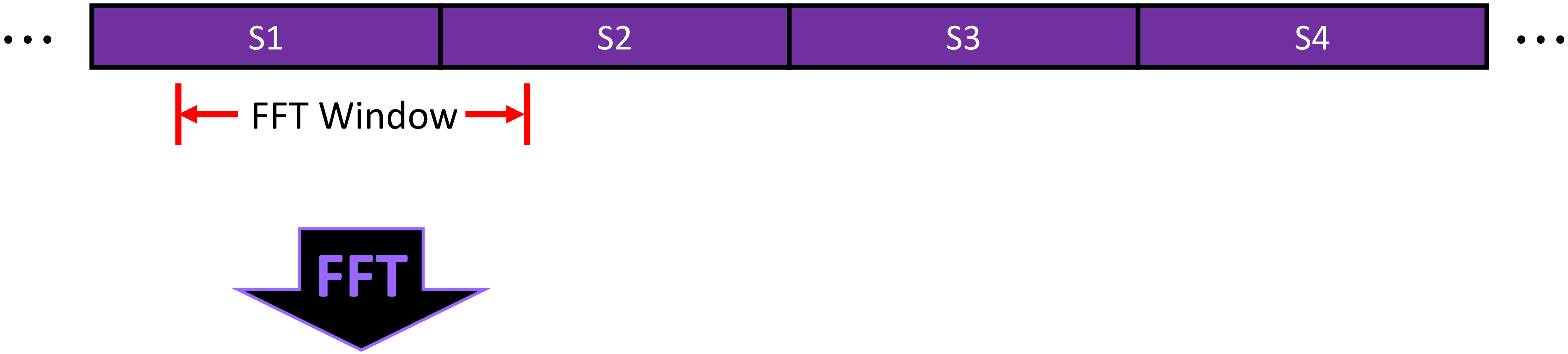
…
S1
S2
S3
S4
…
FFT Window
FFT
FFT
FFT
$\ldots, +1, -1, +1, \ldots$
$\ldots, +1, +1, +1, \ldots$
$\ldots, -1, -1, +1, \ldots$
$\ldots 101 \ldots$
$\ldots 111 \ldots$
$\ldots 001 \ldots$

OFDM Symbol



Assumes FFT window is perfectly aligned with symbol boundaries

OFDM Symbol



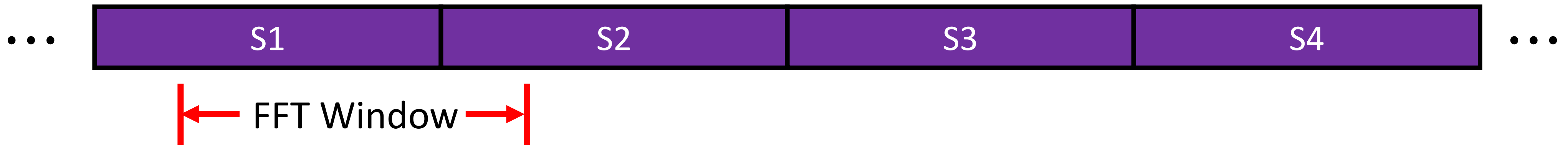
..., +0.5 + 1i, −0.7 + 0.3i, ...

❌ Cannot decode!

FFT window is misaligned with symbol

Subcarriers are no longer orthogonal.

OFDM Cyclic Prefix



- DFT (FFT) assumes time samples are periodic of period N

- Circular Shift before taking FFT:

$$x[t] \rightarrow X[f]$$

$$x[t - \tau \bmod N] \rightarrow X[f]e^{-j\frac{2\pi f \tau}{N}}$$

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OFDM Cyclic Prefix

- Even if FFT window is misaligned, CP ensures that all samples come from the same symbol → Orthogonality is preserved!

- Cyclic Prefix can be created by:
  - Take first few samples and append them to end of symbol.
  - Take last few samples and prefix them to beginning of symbol.

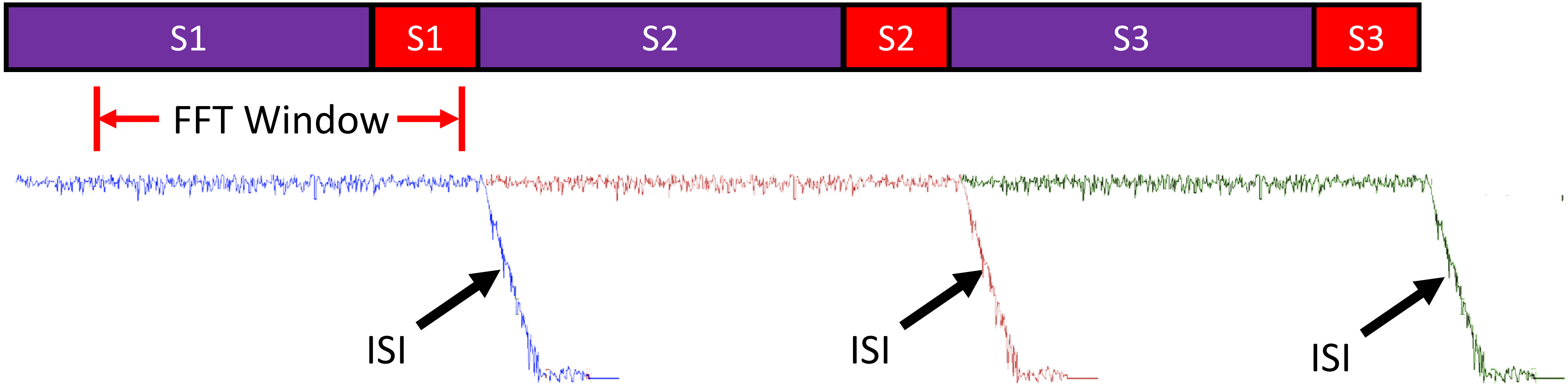
- Simple Phase Shift → Can be corrected by lumping with channel $H[f]$

OFDM Cyclic Prefix

Cyclic Prefix:

- Preserves orthogonality by allowing some misalignment in FFT Window

- Deals with Inter-Symbol-Interference



OFDM Cyclic Prefix

Cyclic Prefix:

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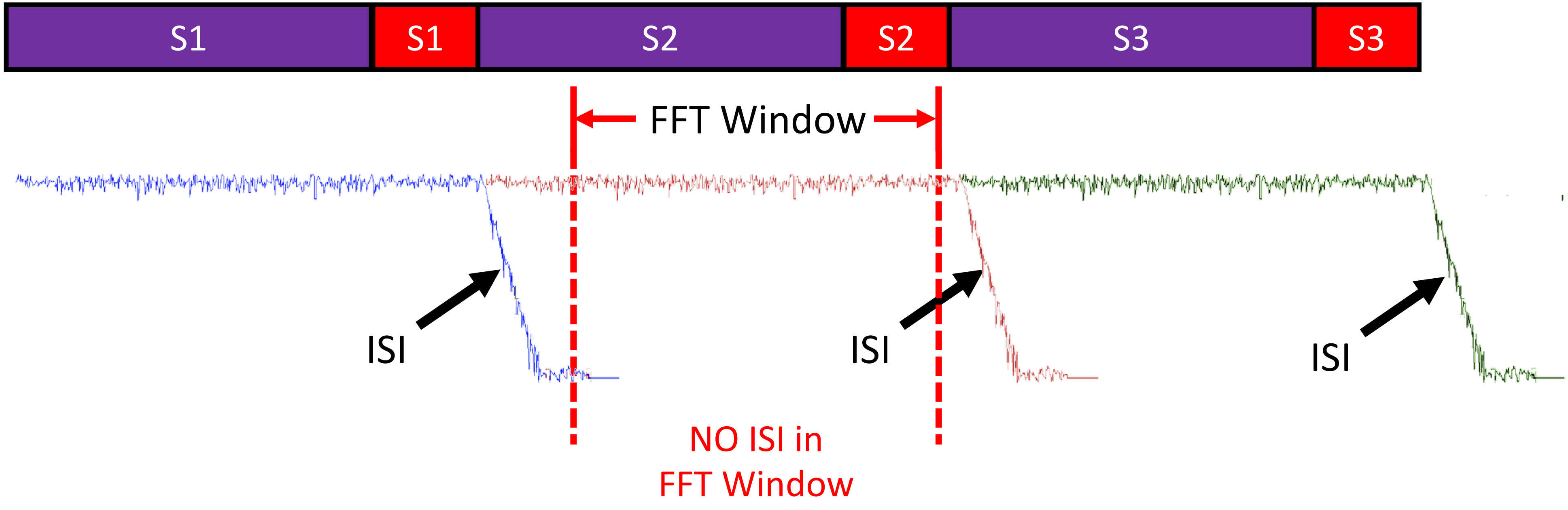

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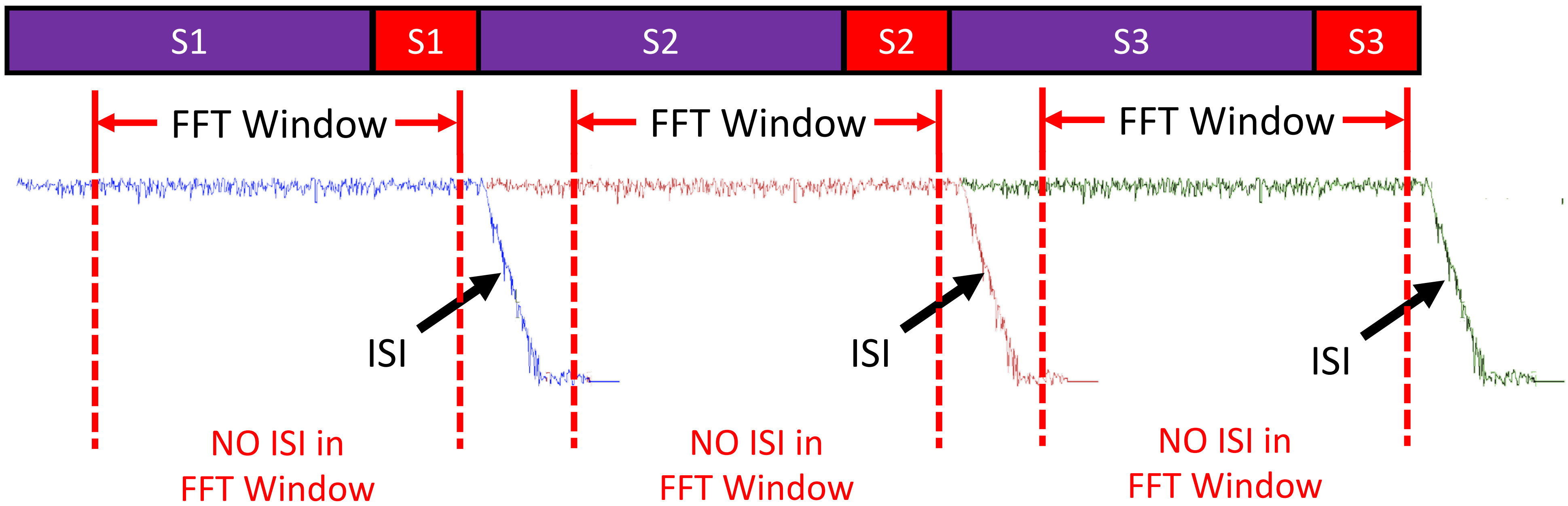

OFDM Cyclic Prefix

Cyclic Prefix:

+ Preserves orthogonality by allowing some misalignment in FFT Window

+ Deals with Inter-Symbol-Interference

− Overhead: Send $CP + N$ samples for every N samples

$$\text{Overhead} = \frac{CP}{CP + N}$$

$$\text{e.g. WiFi 802.11n: } N = 64, \text{CP} = 16 \rightarrow \text{Overhead} = 20\%$$

$$\text{e.g. LTE: } N = 1024, \text{CP} = 72 \rightarrow \text{Overhead} = 6.5\%$$