

ECE 586GT: Problem Set 4

Extensive form (aka sequential) games

Due: Wednesday, October 28, 11:59 p.m.

Reading: Course notes, Sections 5.1 and 5.2

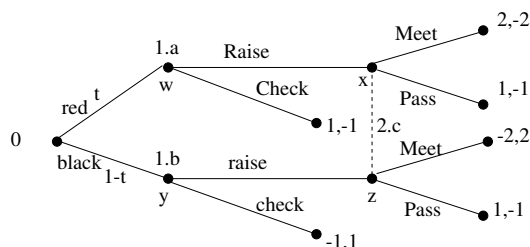
1. [Ultimatum game]

Consider the following two stage game, about how two players split a pile of 100 gold coins. The action (strategy) set of player 1 is given by $S_1 = \{0, \dots, 100\}$, with choice i meaning that player 1 proposes to keep i of the gold coins. Player 2 learns the choice of player one, and then takes one of two actions in response: A (accept) or R (reject). If player two plays accept, the payoff vector is $(i, 100 - i)$. If player two plays reject, the payoff vector is $(0, 0)$.

- Describe the extensive form version of the game using a tree labeled as in class.
- Describe the normal form of the game. It suffices to specify the strategy spaces and payoff functions. (Hint: Player 2 has 2^{101} pure strategies.)
- Identify a Nash equilibria of the normal form game with payoff vector $(50, 50)$.
- Identify the subgame perfect equilibria of the extensive form game. (Hint: There are two of them.)
- Identify the trembling hand perfect pure strategy equilibria of the normal form version of the game.

2. [Call my bluff card game with bias]

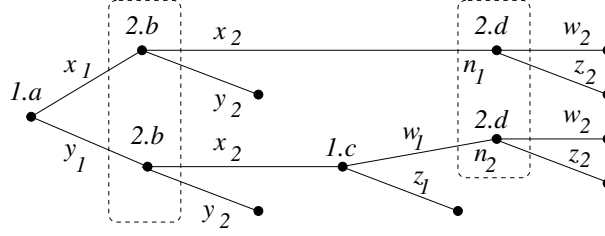
Fix t with $0 < t < 1$ and consider the following version of the call my bluff card game. The special case $t = 0.5$ is equivalent to the call my bluff card game example in the notes.



- Find the expected payoff matrix for player 1 for the normal form version of the game.
- Find all of the Nash equilibria of the game in mixed strategies (including pure Nash equilibria). What is the expected payoff of player 1 at a Nash equilibrium? Your answer should depend on t .
- Find all of the sequential equilibria (σ, μ) for the game. Give explicit expressions for the behavioral strategy profiles σ and the belief vectors μ . Your answer should depend on t .

3. **[Finding a behavioral representation]**

Consider the extensive form game shown, and consider the mixed strategy profile τ for the



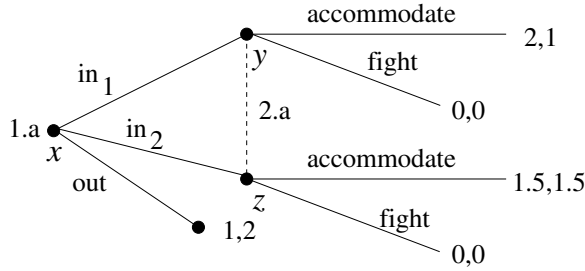
normal representation of the game, given by $\tau = (\tau_1, \tau_2)$ where

$$\begin{aligned}\tau_1 &= .5[x_1w_1] + .5\alpha[y_1w_1] + .5(1 - \alpha)[y_1z_1] \\ \tau_2 &= \beta[x_2w_2] + (1 - \beta)[y_2z_2]\end{aligned}$$

for some constants $\alpha, \beta \in (0, 1)$.

- Does this game satisfy the perfect recall condition? Briefly explain.
 - Find the behavioral representation σ of τ . Does σ_2 depend on τ_1 (i.e. on α)? (Hint: You should specify distributions player 1 uses in information sets 1.a and 1.c, and the distribution player 2 uses in information sets 2.b and 2.d.)
 - Note that n_1 and n_2 are the two nodes in information set 2.d. Find the conditional probability node n_1 is reached given information state 2.d is reached. Does it depend on σ_2 (i.e. on β)?
4. **[Sequential equilibrium for a variation of the entry deterrence game]**

Consider the following variation of the entry deterrence game, which is similar to the variation in Example 5.13 of the notes.



Letters x, y and z in the figure are the names of the three nonleaf nodes, so, for example, the information set 2.a is equal to $\{y, z\}$.

- Give the normal form representation of the game and identify: the Nash equilibrium in pure strategies, the Nash equilibrium in mixed strategies, and the trembling hand perfect equilibrium (in pure or mixed strategies).
- Find the set of all sequential equilibria for this game. Justify your answer. (Hint: By definition, sequential equilibria are assessments that are sequentially rational and consistent.)