Name: _____ **NetID:** _____

ECE 585GT Fall 2026 Exam 1, October 22, 2026

90 min; 1 page notes 2-sided OK; closed book; no calculators

1. **[30 points]** Consider the two-player game with payoff matrix shown, where player 1 selects a letter for a row and player 2 selects a number for a column:

	1	2	3
A	6,4	7,7	4,3
B	8,5	5,2	6,4
C	7,6	3,4	5,5

   (a) (5 points) Identify all dominant strategies and all weakly dominant strategies by either player.

   (b) (5 points) Identify all pairs of strategies such that both strategies are for the same player and one strategy dominates or weakly dominates the other.

   (c) (5 points) Identify the pure Nash equilibria.

   (d) (5 points) Identify the Nash equilibria involving nondegenerate mixed strategies, if any.

   (e) (5 points) Is the game a potential game? If so, identify the potential function. If not, show why.

   (f) (5 points) Consider the iterated best response algorithm. Suppose player 1 selects one of the three pure strategies, all three having equal probability. Then player 2 selects a best response to player 1. Then player 1 selects a best response to player 2. and so on. With what probability does the algorithm converge? Given the algorithm converges, what is the probability distribution of the limiting strategy pair?

2. **[20 points]** Determine whether each statement is TRUE or FALSE, *and give a justification for your answer* for more than half credit.

   (a) (5 points) Consider a finite two-person zero-sum game with payoff matrix A. Suppose V is the value of the game: $V = \min_p \max_q p^T A q = \max_q \min_p p^T A q$. Suppose (p^*, q^*) is a strategy profile such that $V = (p^*)^T A q^*$. Then (p^*, q^*) is a saddle point (aka Nash equilibrium).

   (b) (5 points) There exists at least one minimizer x^* of the function $f : \mathbb{R}^2 \to \mathbb{R}$ such that $f(x_1, x_2) = x_1^4 - 12 x_1 x_2^2 + e^{x_1^2 + 3x_2^2}$.

   (c) (5 points) For a finite normal form game, the set of correlated equilibria is a convex set.

   (d) (5 points) For a player i in a finite normal form game, the set of maxmin strategies is a convex set.

3. **[20 points]** Recall the replicator dynamics equation for the fractional sizes of subpopulations for a given finite set of types S and fitness matrix $(u(x,y))_{x,y \in S}$, where $u(x,y)$ is the fitness of a type x individual encountering a type y individual:

$$\dot{\theta}_t(a) = \theta_t(a)(u(a, \theta_t) - u(\theta_t, \theta_t)).$$

   Let $\overline{\theta}$ be a probability distribution vector indexed by S, representing a distribution of relative population sizes.

(a) (10 points) Prove or disprove the following statement: If $(\overline{\theta}, \overline{\theta})$ is a Nash equilibrium for the symmetric two player game based on u, then $\overline{\theta}$ is a steady state (aka equilibrium) for the population dynamics.

(b) (10 points) Prove or disprove the following statement: If $\overline{\theta}$ is a steady state for the population dynamics then $(\overline{\theta}, \overline{\theta})$ is a Nash equilibrium for the two player symmetric game based on u.

4. **[30 points]** Consider the following two-player game with the two-dimensional payoff function $\ell(i, j)$ for $i, j \in \{0, 1\}$ shown:

	1	2
1	$\binom{0}{0}$	$\binom{1}{0}$
2	$\binom{0}{1}$	$\binom{0}{0}$

(a) (15 points) Consider a halfspace of the form $\mathcal{H} = \{v \in \mathbb{R}^2 : u \cdot v \leq c\}$ such that $u = \binom{u_1}{u_2}$ is a nonzero vector with $u_1 \geq 0$ and $u_2 \geq 0$. What is the minimum value of c such that $\mathcal{H}$ is approachable (in the sense of Blackwell) by player 1? Express c in terms of u as explicitly as possible.

(b) (15 points) What is the minimum value of $\delta \geq 0$ such that the square region $S_\delta = [0, \delta]^2$ is approachable by player 1. Give an explicit numerical value and show your reasoning.