University of Illinois Fall 2017

ECE 586GT: Exam I

Monday October 24, 2017
7:00 p.m. — 8:30 p.m.
2013 Electrical Engineering Building

1. **[15 points]** For each of the following functions, state whether there exists an x^* in the domain of the function that maximizes the function. Justify your answers.
   (a) $f : [0,1]^2 \to \mathbb{R}$ such that $f(x_1, x_2) = x_1 \mathbf{1}_{\{x_1+x_2 \leq 1\}}$.
   (b) $f : [0,1]^2 \to \mathbb{R}$ such that $f(x_1, x_2) = x_1^3 + 12x_1x_2^2 - e^{2x_1^2+x_2^2}$.
   (c) $f : \mathbb{R}^2 \to \mathbb{R}$ such that $f(x_1, x_2) = x_1^3 + 12x_1x_2^2 - e^{2x_1^2+x_2^2}$. (Same form as in (b) but the domain is different.)

2. **[30 points]** Consider the two-player strategic game with the following payoff matrix, where player 1 selects a row and player 2 selects a column:

	1	2	3
1	6,6	3,4	4,3
2	7,3	9,8	9,7
3	3,3	8,8	6,7

.

   (a) (5 points) Identify all dominant pure strategies by either player.
   (b) (5 points) For any player that does not have a dominant strategy, identify all pairs of strategies for that player such that one strategy dominates the other.
   (c) (5 points) Identify the pure strategy Nash equilibria (NE).
   (d) (5 points) Identify the NE involving nondegenerate mixed strategies, if any.
   (e) (5 points) Is the game a potential game? If so, identify the potential function. If not, show why.
   (f) (5 points) Suppose for some initial strategy profile (s_1^0, s_2^0) the players one at a time change their strategies to a best response to the strategy of the other player (i.e. they implement alternating best response). Does the sequence necessarily converge to a Nash equilbrium for this game? Justify your answer.

3. **[30 points]** Consider Shapley's version of the rock-scissors-paper game:

	R	S	P
R	$0,0$	$1,0$	$0,1$
S	$0,1$	$0,0$	$1,0$
P	$1,0$	$0,1$	$0,0$

   (a) (10 points) Find the maximum expected sum of payoffs over all correlated equilibria.
   (b) (10 points) Suppose the game is played repeatedly, and player 1 uses a Hannan consistent strategy. What is the long term minimum average payoff guaranteed to player 1 with probabilty one, as the number of plays converges to infinity?
   (c) (10 points) Is $p^* = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ an evolutionarily stable state (ESS) for the evolutionary game based on the above symmetric two player game? Justify your answer.

4. **[25 points]** Consider a two person zero sum game with $S_1 = S_2 = [0, 1]$ and $\ell(x, y) = (x-y)^2$. As usual, player 1 is a minimizer and player 2 is a maximizer, so ℓ is a loss function for player 1 and a payoff function for player 2. Pure strategies are considered to be special cases of mixed strategies.

   (a) (5 points) Given player 2 uses a mixed strategy q, what strategy of player 1 minimizes the expected loss for player 1? (Hint: Think of q as the distribution of a random variable Y representing the action of player 2.)

   (b) (5 points) Find all maxmin mixed strategies of player 2, and the resulting maxmin value for player 2.

   (c) (5 points) Given player 1 uses a mixed strategy p, what strategy of player 2 maximizes the expected reward for player 2? (Hint: Think of p as the distribution of a random variable X representing the action of player 1.)

   (d) (5 points) Find all minmax optimal strategies for player 1, that is, all mixed strategies of player 1 that minimize the maximum loss for player 1, and find the resulting minmax value for player 1.

   (e) (5 points) Specify all the saddlepoints for the game in mixed strategies and give the value of the game.