University of Illinois Spring 2013

ECE 586: Exam II

Wednesday, May 1, 2013
7:00 p.m. — 8:30 p.m.
168 Everitt Laboratory

1. **[20 points]** (One-step deviations in a repeated game) Consider a two player repeated game such that the strategy space of the stage game is $\{x, y\}$ for both players. An action sequence for the repeated game is denoted by $a = ((a_1(t), a_2(t)) : t \geq 0)$ and for $t \geq 0$ the corresponding history at time t is $h_t = ((a_1(s), a_2(s)) : 0 \leq s < t)$. Consider the strategy profile $\mu = (\mu_1, \mu_2)$ defined as follows.

$$\mu_1(t, h_t) = \begin{cases} x & t \in \{0, 1\} \\ a_2(t-2) & t \geq 2 \end{cases} \qquad \mu_2(t, h_t) = \begin{cases} y & t = 0 \\ a_2(t-2) & t \text{ even and } t \geq 1 \\ a_1(t-1) & t \text{ odd} \end{cases}$$

   (a) (6 points) What is the action sequence generated by μ?

   (b) (7 points) Suppose μ_1' is obtained by deviating from μ_1 at only the single (time, history) pair $(\bar{t} = 3, \overline{h}_{\bar{t}} = ((x, y), (x, x), (x, y)))$. What is the action sequence generated by (μ_1', μ_2)?

   (c) (7 points) Suppose μ_2' is obtained by deviating from μ_2 at only the single (time, history) pair $(\bar{t} = 2, \overline{h}_{\bar{t}} = ((x, y), (x, x)))$. What is the action sequence generated by (μ_1, μ_2')?

2. **[20 points]** Recall the example of a coalitional game discussed in class (in O&R book) based on a production economy with one capitalist c and a set W of w workers. The capitalist owns a factory that can produce at rate $f(i)$ if there are i workers in the factory, where f is a concave increasing function with $f(0) = 0$. This can be modeled as a coalitional game $\langle N, v \rangle$ with $N = \{c\} \cup W$, and

$$v(S) = \begin{cases} 0 & c \notin S \\ f(|S \cap W|) & c \in S. \end{cases}$$

   Find the vector of payoffs x for this game given by the Shapley value.

3. **[25 points]** Consider a production economy with two capitalists, a and b, and a set W of $2w$ workers, for some positive integer w. Each of the capitalists owns a factory and the two factories separately produce the same commodity. Each worker can work in only one factory. Let $f(i)$ denote the production rate of one factory with i workers, for $0 \leq i \leq 2w$, where $f(0) = 0$ and f is a concave, increasing function.

   (a) (10 points) This economy can be modeled as a coalitional game $\langle N, v \rangle$. Specify how the set N and characteristic function v should be defined in this case.

   (b) (15 points) Identify the core of the game.

4. **[35 points]** (Revenue optimal sequential selling mechanism) Suppose there is a population of n bidders, and one seller with an object to sell. The value of the object to bidder i is X_i, where $X_1, \ldots, X_n$ are independent, and each is exponentially distributed with parameter λ. Each bidder i knows his own value, and knows the probability distribution of the other values. The seller knows the probability distribution of all values.

   This problem addresses the Bayes-Nash implementation of a revenue optimal seller mechanism, subject to the following protocol. The bidders appear before the seller one at a time in the fixed order $1, 2, \ldots, n$. When a bidder appears, he submits a bid to the seller, and, on the basis of the bid, the seller must either accept or reject the bid. If the bid is accepted, the bidder wins the object and makes a payment. If the bid is rejected, the bidder has no further chance to win and the seller moves on to the next bidder. The mechanism is also constrained to be individually rational. The seller has the option of not selling the object to any bidder, in which case the value of the object to the seller is zero.

(a) (5 points) Briefly describe what the revelation principle implies in the context of this game, and why it applies.

(b) (5 points) Calculate the virtual valuation function $\psi(x_i) = x_i - \frac{1-F_i(x_i)}{f_i(x_i)}$ of a buyer i. (By symmetry, answer does not depend on i.)

(c) (10 points) Identify the optimal mechanism in this context for $n = 1$, and calculate the resulting expected revenue R_1^*.

(d) (15 points) Let R_n^* denote the maximum expected revenue for the game with n bidders. Describe the actions of the seller for the first bidder in terms of R_{n-1}^*, and find a recursion expressing R_n^* in terms of R_{n-1}^*. *(You can write your answer to this part on the next page.)*