CS 579. Computational Complexity
Problem Set 3

Due May 1, 2016 by midnight

Problem 1

(30 pts.) Consider the following two definitions of log-space counting problems. A function $f : \{0,1\}^* \rightarrow \mathbb{N}$ is in $\#L1$ if there is a non-deterministic Turing machine M_f that on input x of length n uses $O(\log n)$ space and is such that the number of accepting paths of $M_f(x)$ equals $f(x)$. A function $f : \{0,1\}^* \rightarrow \mathbb{N}$ is in $\#L2$ if there is a relation $R(.,.)$ that is decidable in log-space and a polynomial p such that if $R(x,y)$ then $|y| \leq p(|x|)$ and such that $f(x)$ equals $|y : R(x,y)|$. Prove that all functions in $\#L1$ can be computed in polynomial time, while $\#L2$ equals $\#P$.

Problem 2

(30 pts.) Alice and Bob share an arbitrarily long common string S. Alice is given as input a random bit x_A and Bob a random bit x_B. Without communicating with each other, Alice and Bob wish to output bits a and b respectively such that $x_A \wedge x_B = a \oplus b$. Prove that any protocol that Alice and Bob follow has success probability at most 3/4.

Problem 3

Recall that if G is a d-regular graph with transition matrix M, then G^k is the d^k -regular graph with transition matrix M^k that has one edge for each path of length k in G (with repetitions).

- (30 pts.) Prove that if $h(G) \geq \epsilon$, then there is a $k = k(\epsilon)$ that depends only on ϵ and not on the size of G such that $h(G^k) \geq 1/10$.
- (10 pts.) Provide a counterexample to the following statement:

  $$h(G^2) \geq \min\{1/10, 1.01 \times h(G)\}$$

  [Note: the statement may be true (its an open question) if G^2 is replaced by G^3.]