CS 579. Computational Complexity
Problem Set 2

Due March 17, 2016 by midnight

Problem 1

(30 pts.) Prove that for every AM[2] protocol for a language L, if the prover and the verifier repeat the protocol k times in parallel (verifier runs k independent random strings for each message) and the verifier accepts if all k copies accept, then the probability that the verifier accepts $x \notin L$ is at most $(1/3)^k$. Note that you cannot assume that the prover is acting independently in each execution. (Use definition 8.6 for IP from Arora Barak).

Problem 2

(30 pts.) Define a language L to be downward-self-reducible if there is a polynomial time algorithm R that for any n and $x \in \{0,1\}^n$, $R^{L_{n-1}}(x) = L(x)$ where by L_k we denote an oracle that solves L on inputs of size at most k. Prove that if L is downward-self-reducible then $L \in PSPACE$.

Problem 3

Recall that the trace of a matrix A, denoted $tr(A)$ is the sum of the entries along its diagonal.

- (10 pts.) Prove that if an $n \times n$ matrix A has eigenvalues $\lambda_1, \dots, \lambda_n$, then $tr(A) = \sum_{i=1}^{n} \lambda_i$.
- (10 pts.) Prove that if A is a random walk matrix of an n-vertex graph G and $k \geq 1$, then $tr(A^k)$ is equal to n times the probability that if we select a vertex i uniformly at random and take a k step random walk from i, then we end up back in i.
- (10 pts.) Prove that for every d-regular graph G, $k \in \mathbb{N}$ and vertex i of G, the probability that a path of length k from i ends up back in i is at least as large as the corresponding probability in T_d, where T_d is the complete $(d-1)$-ary tree

of depth k rooted at i. (that is, every internal vertex has degree d, one parent and $d-1$ children.)

- (10 pts.) Prove that for even k, the probability that a path of lengh k from the root v of T_d ends up back at v is at least $2^{k-k\log(d-1)/2+o(k)}$.