CS 579. Computational Complexity
Problem Set 1

Due February 16, 2016 by midnight

Problem 1

(10 pts.) Show that $SPACE(n) \neq NP$.

Problem 2

Recall that $E = DTIME(2^{O(n)})$ is the class of problems solvable by deterministic turing machine in time $2^{O(n)}$, where n is the length of the input. We say that a language A has a many-to-one polynomial time reduction to a language B, written $A \leq_m^p B$ if there is a polynomial time computable function $f(\cdot)$ such that for every instance x we have $x \in A \Leftrightarrow f(x) \in B$.

- (10 pts.) Show that NP is closed under polynomial many-to-one reductions, that is $A \leq_m^p B$ and $B \in NP$ implies $A \in NP$.
- (10 pts.) Show that if E were closed under many-to-one reductions, we would have a contradiction to the time hierarchy theorem. Conclude that $NP \neq E$.

Problem 3

(20 pts.) Define a language L which belongs to SIZE($O(1)$) and is undecidable.

Problem 4

Recall that $EXP = DTIME(2^{n^{O(1)}})$.

- (15 pts.) Prove that if $P = NP$ then $\Sigma_k = P$ for all k.
- (15 pts.) Prove that if $P = NP$ then there is a problem in EXP that requires circuits of size $2^{\Omega(n)}$.

Problem 5

(20 pts.) Prove that if $NP \subseteq BPP$ then $NP = RP$.