

Information Theory: Homework II

September 22nd 2025

Problem 1: Properties of mutual information. Let X, Y_1, Y_2 be three RVs.

- (a) Given $I(X; Y_1) = I(X; Y_2) = 0$, does it follow that $I(X; Y_1, Y_2) = 0$?
- (b) Given $I(X; Y_1) = I(X; Y_2) = 0$, does it follow that $I(Y_1; Y_2) = 0$?

Problem 2: Data Processing Inequality. Let the RVs X, U, Z form a Markov chain, $X \rightarrow Y \rightarrow Z$.

- (a) Show that $H(X|Y) = H(X|Y, Z)$.
- (b) Show that $H(X|Y) \leq H(X|Z)$.
- (c) Show that $I(X; Y) \geq I(X; Z)$.
- (d) Show that $I(X; Z|Y) = 0$.

Problem 3: Divergence. Let $d(p||q)$ stand for the KL divergence between two Bernoulli distributions with parameters p and q . Show that for all $p, q \in [0, 1]$, $d(p||q) \geq 2(p - q)^2 \log(e)$. This problem is from Prof. Polyanskiy's course HWs (MIT).

Problem 4: Cover, Thomas, AEP section problem: "An AEP-like limit."

Problem 5: Cover, Thomas, AEP section problem: "Random box size."

Problem 6: Cover, Thomas, Data compression section problem: "How many fingers does a Martian have?"

Problem 7: Depth constraint Huffman codes. Please read the paper "Near-Optimal Depth-Constrained Codes," by Gupta, Prabhakar and Boyd. Describe the main results, the approaches used to establish them and the intuition behind them.