ECE 563: FALL 2024
HOMEWORK I
ISSUED: 6TH OF SEPTEMBER. DUE SEPTEMBER 16TH, AT THE BEGINNING OF THE LECTURE.

Note: The HW is due in class, before the lecture starts. Make sure to be on time as turning the homework in while the lecture is in progress may be distracting for the instructor and your classmates.

- **Problem 1.** Find the entropy of a geometric random variable X with parameter p. Next, assume that Y is the number of coin flips until the second head appears. Prove that $H(Y) \leqslant 2H(X)$ and explain intuitively why this is the case.
- **Problem 2.** An urn contains b blue, g green and r red balls. Does drawing two balls from the urn with replacement have higher entropy than drawing two balls without replacement? Justify your answer.
- **Problem 3.** Cover and Thomas, problem 2.11, *Measure of correlation*.
- **Problem 4.** Solve the following two problems:
  - Find the mutual information between the top and bottom side of a flipped fair coin.
  - Find the mutual information between the top and bottom side of a rolled fair die which has exactly five sides.
- **Problem 5.** Let $w_i, i \in \{1, 2, .., M\}$ be a discrete probability distribution. Prove that
$$\sum_{i=1}^{M} \frac{w_i}{x_i} \geqslant \frac{1}{\sum_{i=1}^{M} w_i x_i},$$
for any collection of positive numbers $x_1, x_2, \ldots x_M$.
- **Problem 6.** Please read the short paper on improving Jensen's inequality, available at https://arxiv.org/pdf/1707.08644 and write a short summary of the results and the examples.
- **Problem 7.** The Rényi entropy of order $\alpha \geqslant 0,\ \alpha \neq 1$ of a discrete RV X supported on a set of cardinality M is defined as
$$H_\alpha(X) = \frac{1}{1-\alpha} \log \left(\sum_{i}^{M} p_i^\alpha \right).$$
  1. Show that $H_0 \geqslant H_1 \geqslant H_2 \geqslant H_\infty$. Observe that the subscripts 1 and ∞ are to be taken in the sense of a limit (i.e., $\alpha \to 1,\ \alpha \to \infty$, respectively).
  2. Show that Rényi entropy is nonnegative, and that it is concave for $\alpha \leqslant 1$.