ECE 563: FALL 2022
HOMEWORK I
ISSUED: 6TH OF SEPTEMBER. DUE SEPTEMBER 15TH, AT THE BEGINNING OF THE LECTURE.

Note: The HW is due in class, before the lecture starts. Make sure to be on time as turning the homework in while the lecture is in progress may be distracting for the instructor and your classmates.

- **Problem 1.** Cover and Thomas, problem 2.13, Ch. 2, "Inequality."
- **Problem 2.** Cover and Thomas, problem 2.19, Ch. 2, "Infinite entropy."
- **Problem 3.** Cover and Thomas, problem 2.21, Ch. 2, "Markov's inequality."
- **Problem 4.** Read the statement and proof of Han's inequality from Polyansky and Wu, page 16. Write the statement of the inequality and prove it on your own after the initial reading. Please do not copy the text directly, try to explain things your way.
- **Problem 5.** Three squares have average area $\bar{a} = 100m^2$. The average of the lengths of their sides is $\bar{l} = 10m$. What can be said about the area of the larges square?
- **Problem 6.** The Rényi entropy of order $\alpha \geqslant 0,\ \alpha \neq 1$ of a discrete RV X supported on a set of cardinality M is defined as

$$H_\alpha(X) = \frac{1}{1-\alpha} \log \left(\sum_i^M p_i^\alpha \right).$$

  1. Show that $H_0 \geqslant H_1 \geqslant H_2 \geqslant H_\infty$. Observe that the subscripts 1 and ∞ are to be taken in the sense of a limit (i.e., $\alpha \to 1,\ \alpha \to \infty$, respectively).
  2. Show that Rényi entropy is nonnegative, and that it is concave for $\alpha \leqslant 1$.