Problem 1: Two-channel analysis.

- Consider two discrete memoryless channels \((X_1, p(y_1|x_1), Y_1)\) and \((X_2, p(y_2|x_2), Y_2)\) with capacities \(C_1\) and \(C_2\), respectively. A new channel \((X_1 \times X_2, p(y_1|x_1) \times p(y_2|x_2), Y_1 \times Y_2)\) is formed in which \(x_1 \in X_1\) and \(x_2 \in X_2\), are simultaneously sent, resulting in \((y_1, y_2)\). Find the capacity of this channel.
- Find the capacity \(C\) of the union of the original two channels where, at each time, one can send a symbol over channel 1 or channel 2 but not both. Assume the output alphabets are distinct and do not intersect. Show that \(2^C = 2^{C_1} + 2^{C_2}\).

Problem 2: Suppose a binary symmetric channel of capacity \(C_1\) is immediately followed by a binary erasure channel of capacity \(C_2\). Find capacity \(C\) of the resulting channel.

Problem 3: Problems 10.1, 10.3, 10.4 from the text.

Problem 4: We are given a set of \(k\) parallel independent additive Gaussian noise channels with noise variances \(N_1, \ldots, N_k\), respectively. A single transmitter is permitted to communicate to a single receiver over this set of channels. The transmitter is power constrained to \(P\). Find the capacity of the system (in bits per use) in each of the following scenarios:

- The transmitter can distribute its available power among the \(k\) channels in any way it likes and can choose the inputs to each channel in any way it likes (as a function of the message it wants to send) subject to the power constraints determined by the way it distributes power over the channels. The receiver receives information from each of the \(k\) channels separately.
- The transmitter is constrained to use exactly the same input in each of the \(k\) channels (as a function of the message it wants to send). The receiver receives information from each of the \(k\) channels separately.
- The transmitter can distribute its available power among the \(k\) channels in any way it likes. The inputs to each channel have to be scaled versions of a single input (as a function of the message the transmitter wishes to send) and subject to the individual power constraints determined by the way the transmitter distributes power over the channels. The receiver receives information from each of the \(k\) channels separately.
- The transmitter is constrained to use exactly the same input in each of the \(k\) channels (as a function of the message it wants to send). The receiver, however, only sees the sum of the outputs of the \(k\) channels.

Problem 5: A memoryless source \(U\) is uniformly distributed on \([0, \ldots, r - 1]\). The distortion function is given as:

\[
d(u, v) = 0, \text{ if } u = v, \quad d(u, v) = 1, \text{ if } u = v \pm 1, \mod r, \quad d(u, v) = \infty, \text{ otherwise.}
\]

Find the rate-distortion function of the source.