

## Lecture 17: Nasal Consonants

Mark Hasegawa-Johnson

ECE 537: Speech Processing

- 1 Impedance, admittance, reactance and susceptance
- 2 Susceptance of a tube open at the other end
- 3 Susceptance of a tube open at the other end
- 4 Formants and antiformants
- 5 Summary

# Outline

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# Review: Solutions to the 1D Acoustic Wave Equation

The 1D acoustic wave equation is solved by forward-going and backward-going waves:

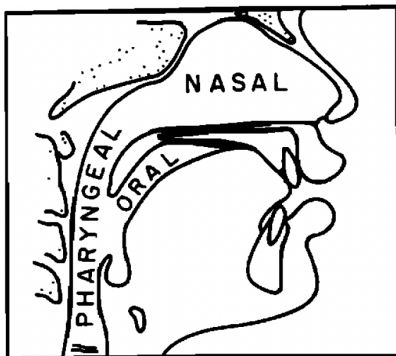
$$u(x, t) = u_+(t - x/c) - u_-(t + x/c)$$

$$p(x, t) = \frac{\rho c}{A} (u_+(t - x/c) + u_-(t + x/c))$$

whose Fourier transforms are:

$$U(x, j\Omega) = U_+(j\Omega)e^{-j\Omega x/c} - U_-(j\Omega)e^{j\Omega x/c}$$

$$P(x, j\Omega) = \frac{\rho c}{A} (U_+(j\Omega)e^{-j\Omega x/c} + U_-(j\Omega)e^{j\Omega x/c})$$



- The shape of a nasal consonant is complicated.
- Instead, let's pick any spot in the tube, and measure the total admittance there:

$$Y(s) \equiv \frac{U(s)}{P(s)}$$

... where  $s = \sigma + j\Omega$  is complex frequency.

# Impedance, Admittance, Resistance, Reactance, Conductance, and Susceptance

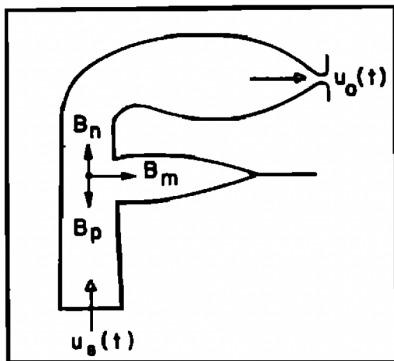
At any point in the tube, we can measure either of:

- Impedance = Resistance + Reactance

$$Z(x, s) \equiv \frac{P(x, s)}{U(x, s)} = R(x, s) + jX(x, s)$$

- ...or Admittance = Conductance + Susceptance

$$Y(x, s) \equiv \frac{U(x, s)}{P(x, s)} = G(x, s) + jB(x, s)$$



A resonant frequency,  
 $s = \sigma + j\Omega$ , is one such that

- ... regardless of how large  $P(x, s)$  may be,
- ... the total flow going into any one tube is matched by flow coming out of the other two tubes, i.e.,

$$Y_p(s) + Y_n(s) + Y_m(s) = 0$$

# Lossless systems

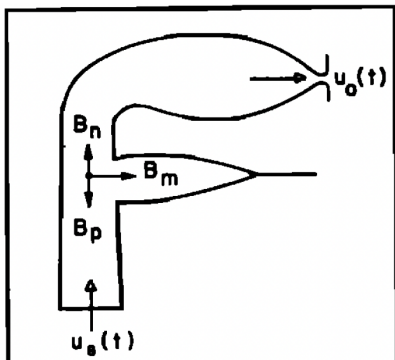
An acoustic system has zero losses if and only if pressure and flow are 90 degrees out of phase everywhere in the system, i.e.,

- Impedance =  $j$  times Reactance

$$Z(x, j\Omega) \equiv \frac{P(x, j\Omega)}{U(x, j\Omega)} = jX(x, j\Omega)$$

- ... Admittance =  $j$  times Susceptance

$$Y(x, j\Omega) \equiv \frac{U(x, j\Omega)}{P(x, j\Omega)} = jB(x, j\Omega)$$



The resonances of a lossless system are at  $s = j\Omega$  such that

- ... regardless of how large  $P(x, j\Omega)$  may be,
- ... the total flow going into any one tube is matched by flow coming out of the other two tubes, i.e.,

$$B_p(j\Omega) + B_n(j\Omega) + B_m(j\Omega) = 0$$

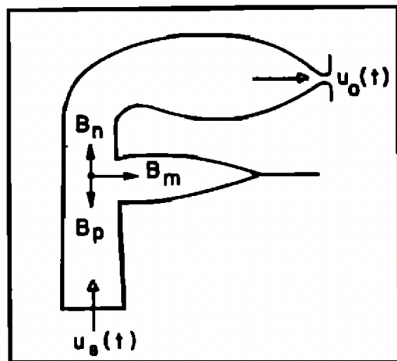
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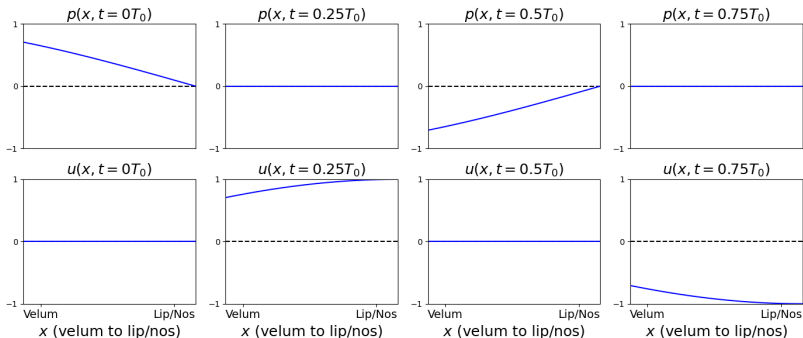
- First consider a tube open at the other end, e.g., the nose.
- At zero frequency, the susceptance is infinite:

$$B_n(0) = \frac{U_n(0)}{P_n(0)} = \frac{\text{finite}}{0} = \pm\infty$$

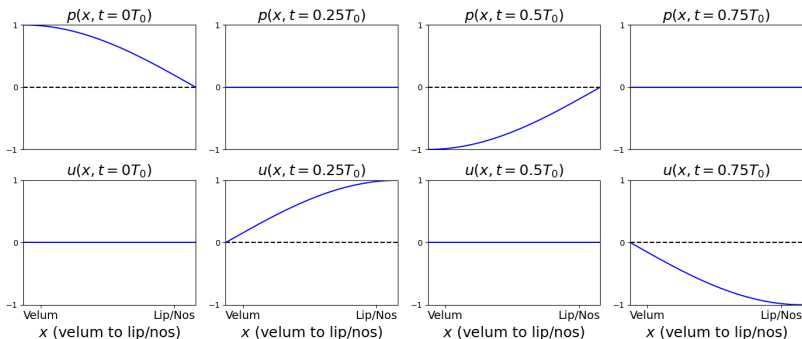
- We don't know yet whether it's  $+\infty$  or  $-\infty$ , we need more information.



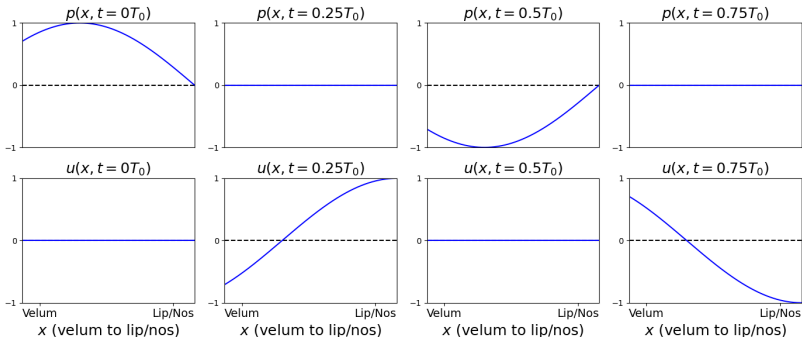
Here are plots of  $u(x, t)$  and  $p(x, t)$ , as functions of  $x$  for  $t \in \left\{ \frac{T_0}{4}, \frac{T_0}{2}, \frac{3T_0}{4} \right\}$ , at the frequency  $f = \frac{c}{8l}$



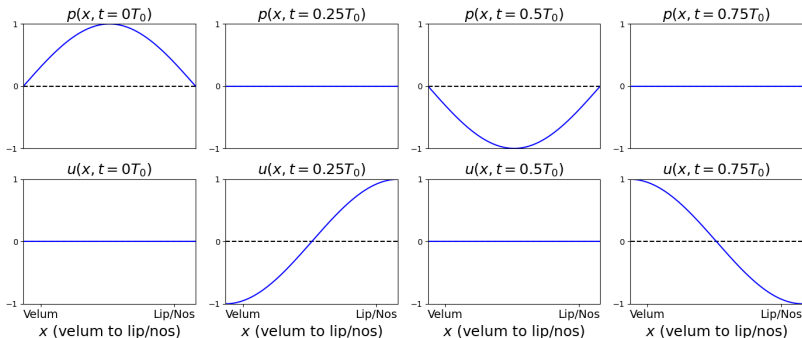
Here are plots of  $u(x, t)$  and  $p(x, t)$ , as functions of  $x$  for  $t \in \left\{ \frac{T_0}{4}, \frac{T_0}{2}, \frac{3T_0}{4} \right\}$ , at the frequency  $f = \frac{c}{4l}$ . At this frequency, no flow enters the left end of the tube, regardless of pressure.



Here are plots of  $u(x, t)$  and  $p(x, t)$ , as functions of  $x$  for  $t \in \left\{ \frac{T_0}{4}, \frac{T_0}{2}, \frac{3T_0}{4} \right\}$ , at the frequency  $f = \frac{3c}{8l}$ .



Here are plots of  $u(x, t)$  and  $p(x, t)$ , as functions of  $x$  for  $t \in \left\{ \frac{T_0}{4}, \frac{T_0}{2}, \frac{3T_0}{4} \right\}$ , at the frequency  $f = \frac{c}{2l}$ . At this frequency, infinite flow can enter even with zero pressure:



# Susceptance of a tube open at the other end

- At low frequencies, pressure accelerates flow, so flow follows  $\frac{T_0}{4}$  seconds ( $\frac{\pi}{2}$  radians) after pressure:

$$Y_n(j\Omega) = \frac{U(j\Omega)}{P(j\Omega)} = jB_n(j\Omega) \propto e^{-j\frac{\pi}{2}} \Rightarrow B_n(j\Omega) < 0$$

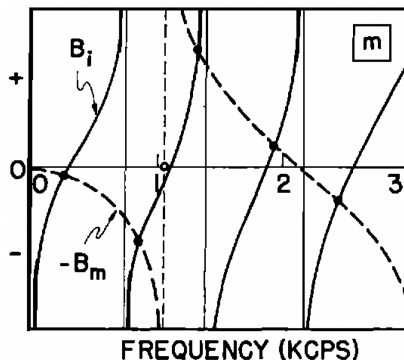
- At frequencies above the susceptance null, positive pressure accelerates a negative flow, so it's as if flow is  $\frac{T_0}{4}$  seconds ( $\frac{\pi}{2}$  radians) ahead of pressure, i.e.,

$$Y_n(j\Omega) = \frac{U(j\Omega)}{P(j\Omega)} = jB_n(j\Omega) \propto e^{+j\frac{\pi}{2}} \Rightarrow B_n(j\Omega) > 0$$

Putting it all together, the susceptance of a tube open at the other end looks like

$B_i = B_n + B_p$  in this figure:

- $B_i(0) = -\infty$
- $B_i(j\Omega)$  is a rising function of  $\Omega$
- $B_i(j\Omega) = \infty$  at  $f = \frac{c}{2l}$ .

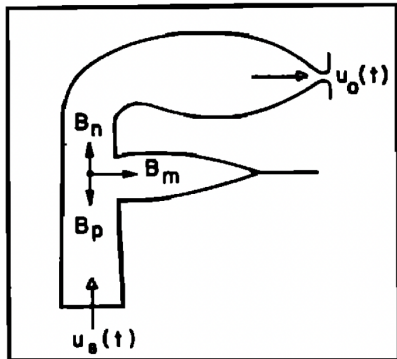


# Outline

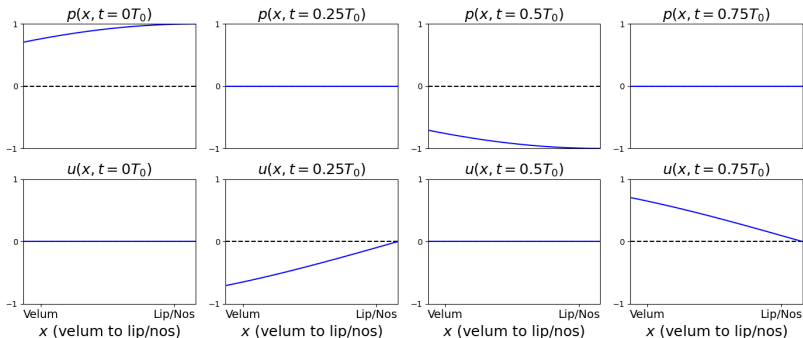
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- Now consider a tube closed at the other end, e.g., the mouth.
- At zero frequency, the susceptance is zero:

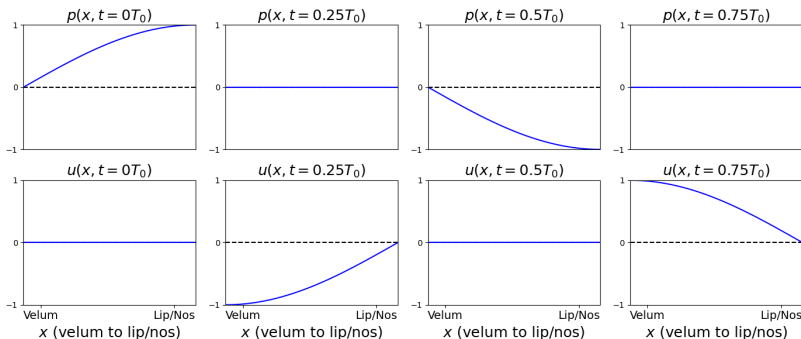
$$B_m(0) = \frac{U_n(0)}{P_n(0)} = \frac{0}{\text{finite}} = 0$$



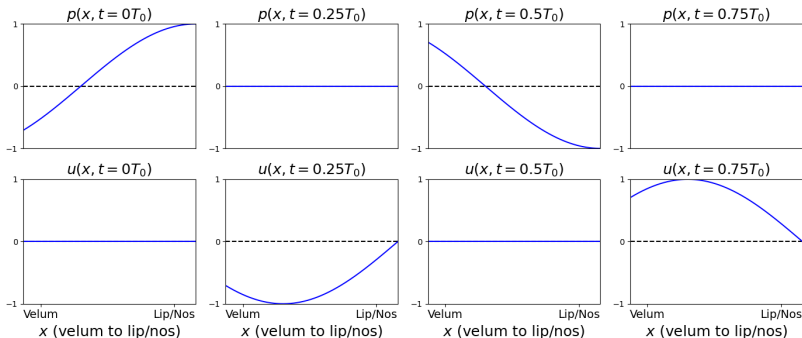
Here are plots of  $u(x, t)$  and  $p(x, t)$ , as functions of  $x$  for  $t \in \left\{0, \frac{T_0}{4}, \frac{T_2}{2}, \frac{3T_0}{4}\right\}$ , at the frequency  $f = \frac{c}{8l}$ .



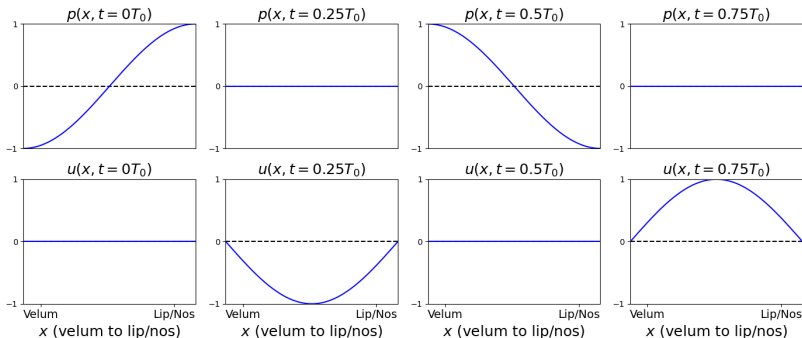
Here are plots of  $u(x, t)$  and  $p(x, t)$ , as functions of  $x$  for  $t \in \left\{ \frac{T_0}{4}, \frac{T_0}{2}, \frac{3T_0}{4} \right\}$ , at the frequency  $f = \frac{c}{4l}$ . At this frequency, infinite flow can enter even with zero pressure:



Here are plots of  $u(x, t)$  and  $p(x, t)$ , as functions of  $x$  for  $t \in \left\{ \frac{T_0}{4}, \frac{T_0}{2}, \frac{3T_0}{4} \right\}$ , at the frequency  $f = \frac{3c}{8l}$ .



Here are plots of  $u(x, t)$  and  $p(x, t)$ , as functions of  $x$  for  $t \in \left\{ \frac{T_0}{4}, \frac{T_0}{2}, \frac{3T_0}{4} \right\}$ , at the frequency  $f = \frac{c}{2l}$ .



# Susceptance of a tube closed at the other end

- At low frequencies, positive pressure accelerates a negative flow, so it's as if flow is  $\frac{T_0}{4}$  seconds ( $\frac{\pi}{2}$  radians) ahead of pressure, i.e.,

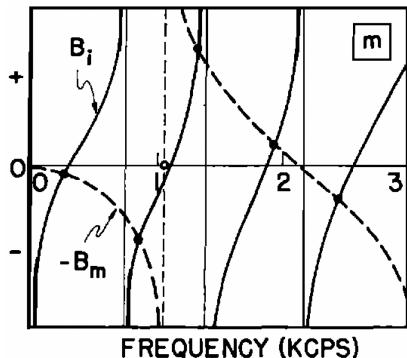
$$Y_n(j\Omega) = \frac{U(j\Omega)}{P(j\Omega)} = jB_n(j\Omega) \propto e^{j\frac{\pi}{2}} \Rightarrow B_n(j\Omega) > 0$$

- At frequencies above the susceptance pole, pressure accelerates flow, so flow follows  $\frac{T_0}{4}$  seconds ( $\frac{\pi}{2}$  radians) after pressure:

$$Y_n(j\Omega) = \frac{U(j\Omega)}{P(j\Omega)} = jB_n(j\Omega) \propto e^{-j\frac{\pi}{2}} \Rightarrow B_n(j\Omega) < 0$$

Putting it all together, the susceptance of a tube closed at the other end looks like  $B_m$  (this figure plots  $-B_m$ ):

- $B_m(0) = 0$
- $B_m(j\Omega)$  is a rising function of  $\Omega$
- $B_m(j\Omega) = \infty$  at  $f = \frac{c}{4l}$
- $B_m(j\Omega)$  is negative at  $f = \frac{3c}{8l}$
- $B_m(j\Omega) = 0$  at  $f = \frac{c}{2l}$



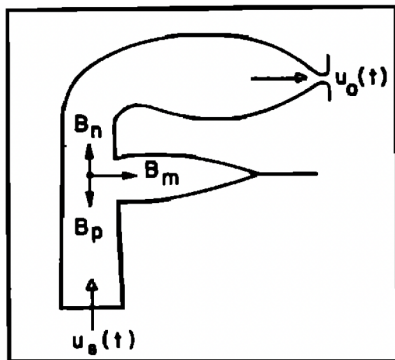
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# Formants and antiformants

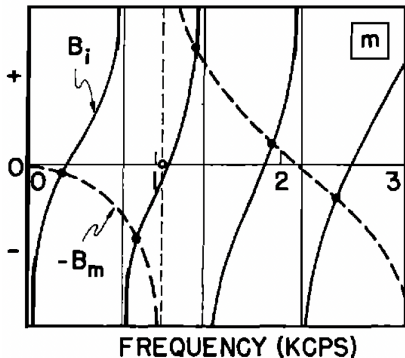
$$B(j\Omega) = \frac{U(j\Omega)}{P(j\Omega)}$$

- If  $B_m(j\Omega) + B_n(j\Omega) + B_p(j\Omega) = 0$ , then  $\Omega$  is a resonance. For an arbitrarily large amount of pressure, the flow out of one tube will just go into another, so there is no loss of mass.
- If  $B_m(j\Omega) = \infty$ , then  $\Omega$  is an antiresonance. Any flow coming out of the nose or pharynx at that frequency will be completely absorbed by the mouth cavity.
- The consonant /ŋ/ has a zero-length mouth cavity, so  $B_m(j\Omega) = \infty$  for that consonant. Its formants are given by  $B_n(j\Omega) + B_p(j\Omega) = 0$ .

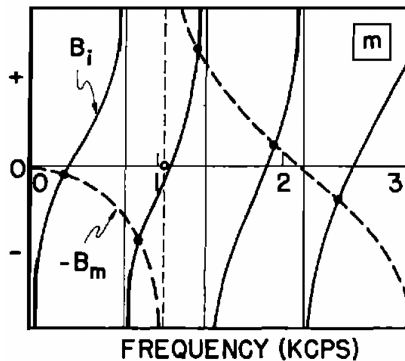


- If  $B_m(j\Omega) + B_n(j\Omega) + B_p(j\Omega) = 0$ , then  $\Omega$  is a resonance.
- If  $B_m(j\Omega) = \infty$ , then  $\Omega$  is an antiresonance ( $\frac{U_o(j\Omega)}{U_s(j\Omega)} = 0$ ).
- The consonant /η/ has a zero-length mouth cavity. Its formants are given by  $B_n(j\Omega) + B_p(j\Omega) = 0$ .

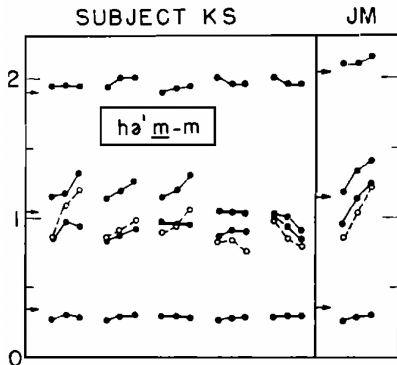
- Here,  
 $B_i(j\Omega) = B_n(j\Omega) + B_p(j\Omega)$ .  
Its zeros are the formants of /η/.
- Open circles show  
 $B_m(j\Omega) = \infty$ , the  
antiformants of /m/.
- Solid dots show  
 $B_i(j\Omega) = -B_m(j\Omega)$ , the  
formants of /m/.



- /ŋ/ has 4 formants.
- /m/ splits the 2<sup>nd</sup> formant into a pole-zero-pole triplet.

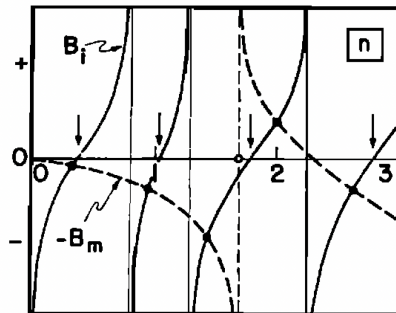


- Here are the formants and antiformants of six different productions of /m/
- /m/ splits the 2<sup>nd</sup> formant of /ŋ/ into a pole-zero-pole triplet.

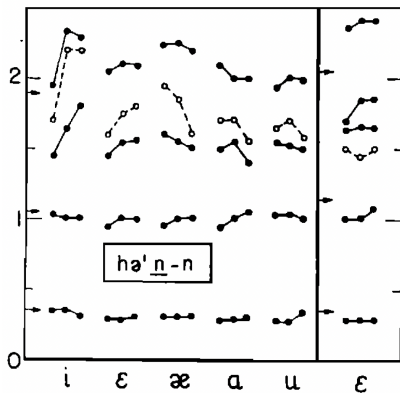


This figure shows  $-B_m(j\Omega)$  of /n/, which has lower slope than that of /m/ because the tube is shorter.

- Because  $B_m(j\Omega)$  is smaller, it splits the 3<sup>rd</sup> formant instead of the 2<sup>nd</sup>.



- Here are the formants and antiformants of six different productions of /n/.
- /n/ splits the 3<sup>rd</sup> formant of /ŋ/ into a pole-zero-pole triplet.



# Formants and antiformants of nasal consonants (approximate averages)

Values from (Fujimura, 1962), rearranged:

|             | /m/           | /n/            | /ɲ/  |
|-------------|---------------|----------------|------|
| F1          | 300           | 300            | 300  |
| B1          | 60            | 40             | 80   |
| F2 or P-Z-P | 900-1000-1100 | 1000           | 1000 |
| B2 or P-Z-P | 60-80-90      | 100            | 100  |
| F3 or P-Z-P | 2000          | 1500-1600-2000 | 2000 |
| B3 or P-Z-P | 280           | 110-600-170    | 230  |

# Big picture: The “nasal murmur”

- Two formants below 1000Hz  $\Rightarrow$  very little energy above 1000Hz
  - Vowels, by contrast, have audible formants all the way up to 3000Hz
  - Fricatives have most of their energy above 2000Hz
- The closure of a nasal consonant is called the “nasal murmur” because most of its energy is at low frequencies (200-1000Hz)
  - Vowels, by contrast, have most of their energy from 200 to 2000Hz
  - Fricatives have most of their energy above 2000Hz

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# Summary

- At any point in the vocal tract, we can measure driving-point impedance or admittance
- Resonant frequencies are those for which the total admittance is zero, i.e.,  $Y_p(s) + Y_n(s) + Y_m(s) = 0$ .
- For  $/\eta/$ ,  $Y_m(s) \approx \infty$ , so formants have  $Y_p(s) + Y_n(s) = 0$ .
- Antiresonances are those for which  $B_m(j\Omega) = 0$