

ECE 537 Homework 5

Due: October 9, Urbana time

1. Sketch a gesture constellation, gestural score, and tract variable score for the word “plum.”

Solution: Onset includes lip closure synchronous with glottal opening, both followed by a tongue tip approximant. Vowel is a tongue body pharyngeal constriction. Coda is a velar opening, which must begin before the beginning of the lip closure.

2. Suppose you have a pipe with a length of l and a cross-sectional area of A , open at both ends. If you blow across one end at the right angle, your breath hitting against the edge of the pipe will generate an air pressure source $P_S(\Omega)$. Assuming the pressure at the other end of the pipe is 0, express the forward-going and backward-going waves in the pipe as functions of $P_S(\Omega)$.

Solution: The boundary conditions are:

$$P_S(\Omega) = \frac{\rho c}{A} (U_+(\Omega) + U_-(\Omega))$$
$$0 = \frac{\rho c}{A} (U_+(\Omega)e^{-j\Omega l/c} + U_-(\Omega)e^{j\Omega l/c})$$

Combining them gives:

$$U_-(\Omega) = -U_+(\Omega)e^{-2j\Omega l/c}$$
$$P_S(\Omega) = \frac{\rho c}{A} U_+(\Omega) (1 - e^{-2j\Omega l/c})$$

which can be solved to give:

$$U_+(\Omega) = \frac{AP_s(\Omega)e^{j\Omega l/c}}{2j\rho c \sin(\Omega l/c)}$$
$$U_-(\Omega) = -\frac{AP_s(\Omega)e^{-j\Omega l/c}}{2j\rho c \sin(\Omega l/c)}$$

3. In class we said that, in the rounded vowels of English (/u, ʊ, o/), the entire vocal tract behind the lips is a single large half-wave resonator. That’s not quite true: besides the labial constriction, these vowels also have a high back tongue body constriction. These vowels can therefore be better approximated with four cavities: cavity 1 is the pharynx, cavity 2 is the velar constriction, cavity 3 is the mouth, cavity 4 is the labial constriction. The lowest formant (F_1) is created by an acoustic compliance with volume $V_b = l_1 A_1 + l_2 A_2 + l_3 A_3$ interacting with an acoustic mass with area A_4 and length l_4 . The second formant (F_2) involves out-of-phase oscillation of the pharynx and mouth, driving air back and forth through the velar constriction, with $u_4(t) \approx 0$ flow escaping through the lips. Write formulas for F_1 and F_2 in terms of c , V_b , and the lengths, areas, or volumes of cavities 1, 2, 3, and/or 4.

Solution: The first formant is governed by

$$M = \frac{\rho l_4}{A_4}$$
$$C = \frac{V_b}{\rho c^2}$$

and has a frequency of

$$\begin{aligned} F_1 &= \frac{1}{2\pi\sqrt{MC}} \\ &= \frac{c}{2\pi} \sqrt{\frac{A_4}{l_4 V_b}} \end{aligned}$$

The second formant has a pressure drop of $p_1(t) - p_3(t)$ across the velar constriction, thus

$$\begin{aligned} p_1(t) - p_3(t) &= \frac{\rho l_2}{A_2} \frac{\partial u_2(t)}{\partial t} \\ P_1(\Omega) - P_3(\Omega) &= j\Omega \frac{\rho l_2}{A_2} U_2(\Omega) \end{aligned}$$

Since flow through the glottis and flow through the lips are both zero, we can approximate

$$\begin{aligned} \frac{\partial u_1(x, t)}{\partial x} &= \frac{u_2(t)}{l_1} \\ \frac{\partial u_3(x, t)}{\partial x} &= -\frac{u_2(t)}{l_3} \end{aligned}$$

and therefore

$$\begin{aligned} -U_2(\Omega) &= j\Omega \frac{V_1}{\rho c^2} P_1(\Omega) \\ &= -j\Omega \frac{V_3}{\rho c^2} P_3(\Omega) \end{aligned}$$

which can be combined to

$$P_1(\Omega) - P_3(\Omega) = -\frac{\rho c^2 U_2(\Omega)}{j\Omega} \left(\frac{1}{V_1} - \frac{1}{V_3} \right)$$

We can simplify these to

$$\begin{aligned} \Delta P(\Omega) &= j\Omega M U_2(\Omega) \\ U_2(\Omega) &= -j\Omega C \Delta P(\Omega) \end{aligned}$$

... where ...

$$\begin{aligned} M &= \frac{\rho l_2}{A_2} \\ C &= -\frac{1}{\rho c^2 (1/V_1 + 1/V_3)} \end{aligned}$$

Thus

$$\begin{aligned} F_2 &= \frac{1}{2\pi\sqrt{MC}} \\ &= \frac{c}{2\pi} \sqrt{\frac{A_2}{l_2} \left(\frac{1}{V_1} + \frac{1}{V_3} \right)} \end{aligned}$$