University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 534: Random Processes

Spring 2005

Problem Set 4

Kalman Filtering, Random Processes, Properties of Random Processes

Issued: Monday, March 14th **Due:** Wednesday, March 30th (beginning of lecture)

Reading from Hajek: Chapter 3, Section 3.5; Chapter 4, Sections 4.1–4.7.
Reading from Stark and Woods: Chapter 6, Section 6.1; Chapter 7, Sections 7.1–7.2.

Problem 4.1

From Hajek, Chapter 3: Problem 8.

Problem 4.2

From Hajek, Chapter 4: Problems 1, 2 and 3.

Problem 4.3

(a) Let $X[n]$ be a discrete-time random process defined for $n = 1, 2, 3, \ldots$. Samples $X[1]$, $X[2]$, $X[3]$, ..., are independent, identically distributed (i.i.d.) random variables and have $\Pr(X[n] = 0) = \Pr(X[n] = 1) = 1/2$. Let the random process $Y[n]$ be defined by

$$Y[n] = X[n] - X[n-1] \; .$$

Find $E[Y[n]]$ and $\mathrm{Var}[Y[n]]$.

(b) Let $X(t)$ be a random process defined by $X(t) = At + B$.

(i) If A is constant and B is a random variable that is uniformly distributed in $[0, 2]$, sketch a sample function of this process and find $E[X(t)]$.

(ii) If A, B are independent, identically distributed (i.i.d.) Gaussian random variables with mean 0 and variance σ^2, find the joint pdf $f_{X(t_1),X(t_2)}(x_1, x_2)$.

Problem 4.4

Let $B[n]$ be a Bernoulli random sequence equally likely to take values ± 1 (independently between different time steps). Define the random process

$$X(t) = \sqrt{p} \sin\left(2\pi f t + B[n]\frac{\pi}{2}\right) \quad \text{for } nT \leq t < (n+1)T \quad \text{for all } n \; ,$$

where $\sqrt{p}$ and f_0 are real numbers.

(a) Determine the mean function $\mu_X(t)$.

(b) Determine the covariance function $K_{XX}(t_1, t_2)$.

Problem 4.5

Let X be a random variable and $Y_0, Y_1, Y_2, ...$ be a sequence of random variables. Show that the random process defined by

$$Z[n] = E[X \mid Y_0, Y_1, ..., Y_n]$$

is a Martingale.

Problem 4.6

A nonuniform Poisson counting process $N(t)$ with time varying rate $\lambda(t)$ ($\lambda(t) \geq 0$ for all $t \geq 0$) is defined for $t \geq 0$ as follows:

(i) $N(0) = 0$;

(ii) $N(t)$ has independent increments;

(iii) For all $t_2 \geq t_1$,

$$P[N(t_2) - N(t_1) = k] = \frac{u^k}{k!}\exp(-u), \quad \text{for } k \geq 0 \ ,$$

where $u = \displaystyle\int_{t_1}^{t_2} \lambda(v)dv$.

Answer the following:

(a) Find $\mu_N(t)$.

(b) Find $R_N(t_1, t_2)$.

(c) Repeat parts (a) and (b) for the case when $\lambda(t) = 1 + 2t$.

Problem 4.7

From Hajek, Chapter 4: Problems 4, 5, 6 and 7.