University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 434: Random Processes

Spring 2004

Problem Set 6

Calculus of Random Processes; Mean Square Continuity, Differentiation and Integration; Ergodicity

Issued: Wednesday, April 14th **Due:** Never (but included in Midsemester Exam 2)

Reading from Hajek: Chapter 5, Sections 5.1–5.5.

Reading from Stark and Woods: Chapter 8, Sections 8.1–8.4.

Problem 6.1

From Hajek, Chapter 5: Problems 3 and 4.

Problem 6.2

Let the random process $X(t)$ be a stationary random process with mean μ_X and covariance function

$$K_{XX}(\tau) = \frac{\sigma_X^2}{1+\alpha^2\tau^2} \ .$$

(a) Show that a mean-square derivative exists for all t.

(b) Find $\mu_{X'}(t)$ and $K_{X'X'}(\tau)$ for all t and τ.

Problem 6.3

Let $X(t)$ be a wide-sense stationary Gaussian random process. Show that the m.s. derivative of

$$Y(t) \equiv X^2(t)$$

is $Y'(t) = 2X(t)X'(t)$ and find the correlation function of $Y'(t)$ in terms of $R_X(\tau)$ and its derivatives. [**Hint:** Recall that for jointly Gaussian random variables $E[X_1X_2X_3X_4] = E[X_1X_2]E[X_3X_4] + E[X_1X_3]E[X_2X_4] + E[X_1X_4]E[X_2X_3]$.]

Problem 6.4

From Hajek, Chapter 5: Problems 5 and 6.

Problem 6.5

This problem concerns ergodicity for random processes.

(a) State the general definition of "ergodic in the mean" for a wide-sense stationary process $X(t)$.

(b) Let $X(t)$ be a wide-sense stationary Gaussian random process with zero-mean and correlation function

$$R_X(\tau) = \sigma^2 e^{-\alpha|\tau|} \cos(2\pi f \tau) \, ,$$

where σ^2, α and f are all constants. Show that $X(t)$ is ergodic in the mean.

(c) Assume that a wide-sense stationary random process $X(t)$ is ergodic in the mean and that the limit of $K_X(\tau)$ as $\tau \to \infty$ exists. Show that

$$\lim_{|\tau| \to \infty} K_X(\tau) = (\mu_X)^2 \, .$$

Problem 6.6 (optional)

From Hajek, Chapter 5: Problem 7.