University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 434: Random Processes

Spring 2004

Problem Set 4

Mean Squared Error Estimation, Kalman Filtering, Random Processes

Issued: Monday, March. 8th **Due:** Never (but included in Midsemester Exam 1)

Reading from Hajek: Chapter 3, Sections 3.4–3.5; Chapter 4, Sections 4.1–4.3, 4.6.

Reading from Stark and Woods: Chapter 6, Section 6.1; Chapter 7, Sections 7.1–7.2.

Problem 4.1

From Hajek, Chapter 3: Problem 7.

Problem 4.2

From Hajek, Chapter 3: Problem 8.

Problem 4.3

From Hajek, Chapter 4: Problems 1, 2 and 3.

Problem 4.4

(a) Let $X[n]$ be a discrete-time random process defined for $n = 1, 2, 3, \ldots$. Samples $X[1]$, $X[2]$, $X[3]$, ..., are independent, identically distributed (i.i.d.) random variables and have $\Pr(X[n] = 0) = \Pr(X[n] = 1) = 1/2$. Let the random process $Y[n]$ be defined by

$$Y[n] = X[n] - X[n-1] \, .$$

Find $E[Y[n]]$ and $\mathrm{Var}[Y[n]]$.

(b) Let $X(t)$ be a random process defined by $X(t) = At + B$.

(i) If A is constant and B is a random variable that is uniformly distributed in $[0, 2]$, sketch a sample function of this process and find $E[X(t)]$.

(ii) If A, B are independent, identically distributed (i.i.d.) Gaussian random variables with mean 0 and variance σ^2, find the joint pdf $f_{X(t_1),X(t_2)}(x_1, x_2)$.

Problem 4.5

From Hajek, Chapter 4: Problems 4 and 5.

Problem 4.6

Let $B[n]$ be a Bernoulli random sequence equally likely to take values ± 1 (independently between different time steps). Define the random process

$$X(t) = \sqrt{p} \sin\left(2\pi f t + B[n]\frac{\pi}{2}\right) \quad \text{for } nT \leq t < (n+1)T \quad \text{for all } n\,,$$

where $\sqrt{p}$ and f_0 are real numbers.

(a) Determine the mean function $\mu_X(t)$.

(b) Determine the covariance function $K_{XX}(t_1, t_2)$.