

ECE 534 Exam 1

Monday October 14, 2013

7:00 p.m. — 8:15 p.m.

124 Burrill Hall

1. **[30 points]** Let V have the exponential distribution with parameter $\lambda = 3$. Determine which of the four sense(s), a.s., m.s., p., or d., that each of the following three sequences of random variables converges, *to a finite limit random variable*.
 - (a) $X_n = \cos\left(\frac{V}{n}\right)$ for $n \geq 1$.
 - (b) $Y_n = \frac{V^n}{n}$ for $n \geq 1$.
 - (c) $Z_n = \left(1 + \frac{V}{n}\right)^n$ for $n \geq 1$.
2. **[20 points]** Let U_1, U_2, \dots be a sequence of independent random variables, each uniformly distributed on the interval $[0, 1]$.
 - (a) For what values of $c \geq 0$ does there exist b with $b > 0$ (depending on c) so that $P\{U_1 + \dots + U_n \geq cn\} \leq e^{-bn}$ for all $n \geq 1$?
 - (b) For what values of $c \geq 0$ does there exist b with $b > 0$ (depending on c) so that $P\{U_1 + \dots + U_n \geq c(U_{n+1} + \dots + U_{2n})\} \leq e^{-bn}$ for all $n \geq 1$?
3. **[30 points]** Let X and Y be random variables with finite fourth moments. For each of the following statements, determine if the statement is true. If true, give a justification. If not true, give a simple counterexample.
 - (a) $E[(X - E[X|Y])^2] \leq E[(X - \hat{E}[X|Y, Y^2])^2]$.
 - (b) $E[(X - E[X|Y^2])^2] \leq E[(X - \hat{E}[X|Y])^2]$.
 - (c) If $E[(X - E[X|Y])^2] = \text{Var}(X)$ then X and Y are uncorrelated.
4. **[20 points]** Let $\begin{pmatrix} X \\ Y_1 \\ Y_2 \end{pmatrix}$ be a mean zero random vector with correlation matrix $\begin{pmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0.5 \\ 0 & 0.5 & 1 \end{pmatrix}$.
The random variable X is to be estimated.
 - (a) Find the linear innovations sequence $(\tilde{Y}_1, \tilde{Y}_2)$ for (Y_1, Y_2) .
 - (b) Either using your answer to part (a) or starting from scratch, identify $\hat{E}[X|Y_1, Y_2]$ explicitly in terms of Y_1 and Y_2 .