ECE 534 RANDOM PROCESSES **FALL 2008**
PROBLEM SET 1 **Due Wednesday, September 10**

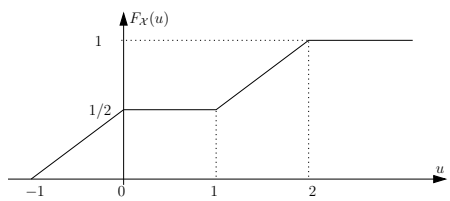
Please visit the course website soon: courses.ece.uiuc.edu/ece534

1. Review of Basic Probability

Assigned reading: Chapter one, "Getting started," of the course notes, including the problems at the end of the chapter. It would also be good to review some of the final exams from the last few semesters of ECE313. The past ECE313 homeworks, exams, and their solutions are available on the ECE 313 website.
Problems to be handed in:

1 Some events based on rolls of three dice
An experiment consists of rolling three fair dice. The rolls are independent trials. Let A be the event that the numbers showing on the three dice are all even, and let B be the event that the numbers showing on the three dice are different from each other. (a) Express the set of possible outcomes Ω using mathematical set notation. (b) Find $P(A)$. (c) Find $P(B)$. (d) Find $P(AB)$. (e) Find $P(A \cup B)$. (f) *Sketch and carefully label* the probability mass function of X, where X denotes the number of distinct numbers showing on the dice.

2 A piecewise linear CDF
Suppose the CDF of a continuous random variable is as shown below.



(a) Find $P\{|X| \leq 0.8\}$. (b) Find $E[X]$. (c) Find $\mathrm{Var}(X)$.

3 Working with first and second moments
Let X be a random variable with mean 4 and variance 16. (a) Find the numerical value of $E[X^2]$. Remember to explain your reasoning. (b) Find the numerical value of $E[(X+2)(X+3)]$. Show your work.

4 A half sine wave pdf
Let X have the pdf $f_X(x) = \begin{cases} \frac{\sin(x)}{2} & x \in [0, \pi] \\ 0 & \text{else} \end{cases}$
(a) Find the cumulative distribution function F_X. In particular, what is $F_X(2\pi)$?
(b) Compute $E[\sin(X)]$.

5 Waiting to see both sides of a biased coin
Consider repeated independent tosses of a biased coin with $P(\text{Heads}) = p$, and let X denote the

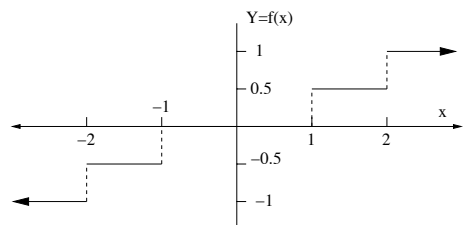
number of tosses required to observe both one Head and one Tail. (a) What is the minimum possible value of X? (b) Find the probability mass function of X. (c) Find the expected value of X. (Find a closed form answer, with no infinite sum.)

6 From moments to probabilities

Suppose X is a random variable with mean 10 and variance 3. Find the numerical value of $P\{X < 10-\sqrt{3}\}$ (or, nearly equivalently, $P\{X < 8.27\}$) for the following two choices of distribution type: (a) Assuming X is a Gaussian random variable. (b) Assuming X is a uniform random variable.

7 Scaler quantization of a Gaussian random variable

Suppose a $N(0, 9)$ random variable is passed through the quantizer function f shown. The output is $Y = f(X)$.



(a) Express the pmf of Y in terms of the Q function. (Check your answer. Make sure your answer is positive for the right values of y.)
(b) Express the variance of Y in terms of the Q function. Give as simple an answer as possible.

8 Working with a power law pdf

Let X be a random variable with pdf $f_X(u) = \begin{cases} ce^u & \text{if } 0 \le u \le 4 \\ 0 & \text{else.} \end{cases}$ For all parts of this problem, you can use terms such as e^4 in your answer. Numerical values are not required. (a) Sketch f_X. (b) Find the constant c. (c) Find $P(X \ge 3 \mid 2 \le X \le 6)$. (d) Find $P(2 \le X \le 6 \mid X \ge 3)$.

9 Folding a pdf

Suppose Z is a *uniform* random variable on the interval $[-2, 4]$. (a) Find $\mathsf{E}[|Z|]$.
(b) Give the pdf of $Y = |Z|$ in equation form (specify it for $-\infty < v < \infty$) *and* sketch the pdf.

10 Using a joint density

Suppose X and Y are jointly continuous random variables distributed over the unit square with the joint pdf given by

$$f_{X,Y}(x, y) = \begin{cases} \frac{3x^2}{2} + 2xy & x, y \in [0, 1] \\ 0 & \text{else} \end{cases}$$

(a) Calculate $E[X]$.
(b) Are X and Y independent? Briefly justify your answer.
(c) Calculate the pdf, $f_Y(y)$, of Y. Be sure to specify it for $-\infty < y < \infty$.
(d) Calculate the conditional density $f_{X|Y}(x|y)$. Be sure to indicate what values of y it is well-defined for, and for such y, specify it for $-\infty < x < \infty$.