University of Illinois at Urbana-Champaign

ECE 534: Random Processes

Fall 2008
Probability Quiz

Monday, September 15

Name: _____

- You have one hour for this quiz. The quiz is closed book and closed note.

- Calculators, laptop computers, Palm Pilots, two-way e-mail pagers, etc. may not be used.

- Write your answers in the spaces provided.

- **Please show all of your work. Answers without appropriate justification will receive very little credit.** If you need extra space, use the back of the previous page.

Score:

1. _____ (12 pts.)

2. _____ (8 pts.)

3. _____ (6 pts.)

4. _____ (8 pts.)

Total: _____(34 pts.)

Problem 1(12 points) Suppose three coins are in a bag. One is double-headed (i.e. a head on each side), one is double-tailed (a tail on each side), and one is a fair coin (head on one side, and tail on the other side). A coin is selected from the bag, with each coin having probability of being selected equal to one third. The coin is then tossed repeatedly, landing heads or tails each time.

(a) What is the probability the first three tosses all land heads?

(b) What is the conditional probability the coin is double-headed, given the first three tosses all land heads?

(c) What is the conditional probability the fourth coin toss lands heads, given the first three tosses all land heads?

Problem 2(8 points) Let X, Y have a joint pdf given by

$$f_{X,Y}(x,y) = \begin{cases} 8x(1-y) & x \geq 0, \ y \geq 0, \ x+y \leq 1 \\ 0 & \text{else.} \end{cases}$$

(a) Find the pdf of Y, f_Y.

(b) Find the conditional pdf of X given Y, $f_{X|Y}(x|y)$. Be sure to indicate for what values of x and y it is well defined.

Problem 3 (6 points) Let X be exponentially distributed with parameter $\lambda = 1$, so $f_X(x) = e^{-x}$ for $x \geq 0$ and $f_X(x) = 0$ for $x < 0$. Let $Z = X^2$. Find the pdf f_Z. Be sure to specify it everywhere.

Problem 4 (8 points) Let $Y = U(1 - U)$ where U is uniformly distributed over the interval $[0, 1]$.
(a) Determine $E[Y]$.

(b) Determine Var(Y).