University of Illinois at Urbana-Champaign

ECE 534: Random Processes

Fall 2008
Exam 2

Monday, November 17, 2008

Name: ____

- You have 75 minutes for this exam. The exam is closed book and closed note, except you may consult both sides of two $8.5'' \times 11''$ sheet of notes.
- Calculators, laptop computers, Palm Pilots, two-way e-mail pagers, etc. may not be used.
- Write your answers in the spaces provided.
- **Please show all of your work. Answers without appropriate justification will receive very little credit.** If you need extra space, use the back of the previous page.

Score:

1. __________ (12 pts.)

2. __________ (12 pts.)

3. __________ (16 pts.)

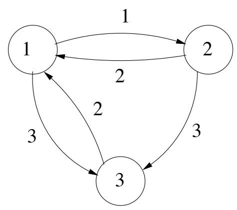
Total: __________(40 pts.)

Problem 1 (12 points) Suppose $a = (a_0, a_1, a_2, a_3)$ is a probability vector to be estimated by observing $Y = (Y_1, T_2, \cdots, Y_T)$. Assume $Y_1, \ldots, Y_T$ are independent, and $P\{Y_t = i\} = a_i$ for $1 \leq t \leq T$ and $i \in \{0, 1, 2, 3\}$.

(a)(6 points) Determine the maximum likelihood estimate, $\widehat{a}_{ML}(y)$, given a particular observation $y = (y_1, \ldots, y_T)$. Justify your answer.

(b)(6 points) Suppose in addition to the above that a has the form $a_i = \binom{3}{i} q^i (1-q)^{3-i}$, or in other words, that a is a binomial pmf with parameters (3,q). Determine the maximum likelihood estimate, $\widehat{q}_{ML}(y)$, given a particular observation $y = (y_1, \ldots, y_T)$. Justify your answer.

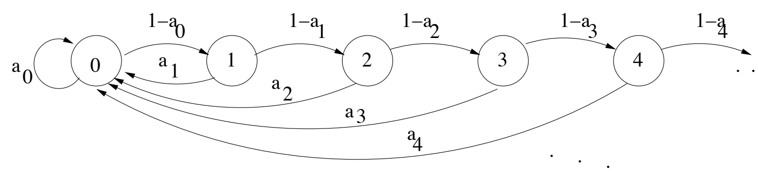
Problem 2 (12 points) Consider a continuous-time Markov process with the rate transition diagram shown:



(a) (2+4 points) Write down the rate transition matrix Q and find the equilibrium distribution π.

(b) (6 points) Find $P[X_t \neq 3 \text{ for } 0 \leq t \leq 5 | X_0 = 1]$. Be as explicit as possible. (Hint: for the particular rates given, this can be solved with very little calculation.)

Problem 3 (16 points) Consider a discrete-time, time-homogeneous Markov process with state space $\mathbb{Z}_+$ and the one-step transition probability diagram shown, where $0 < a_k < 1$ for all $k \geq 0$. Note that this is *not* a birth-death process.



(a)(4 points) Sketch the one-step transition probability matrix. (The matrix has infinitely many entries, so use dot dot dot "$\cdots$" notation.)

(b) (4 points) Under what condition on the a's is the process recurrent?

(c) (4 points) Under what condition on the a's is the process positive recurrent?

(d) (4 points) Find the equilibrium distribution π in terms of the a's, and in particular, indicate when an equilibrium distribution exists. Be as explicit as possible.