

University of Illinois at Urbana-Champaign

ECE 534: RANDOM PROCESSES

Fall 2007

Probability Quiz

Monday, September 10, 2007

Name: \_\_\_\_\_

- This is a closed-book exam. No notes of any kind are allowed.
- Calculators, laptop computers, Palm Pilots, two-way email pagers, etc. may not be used.
- Write your answers in the space provided. If you need extra space, use the back of the previous page.
- Please show all of your work. Answers without appropriate justification will receive very little credit.

Score:

1: \_\_\_\_\_ (12 points)

2: \_\_\_\_\_ (12 points)

3: \_\_\_\_\_ (12 points)

Total: \_\_\_\_\_ (36 points)

1 Suppose continuous RVs  $X$  and  $Y$  have the joint pdf

$$f_{X,Y}(x,y) = \begin{cases} \frac{x}{C} & \text{if } 1 \leq x \leq 3 \text{ and } 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Let  $R$  be defined by  $R = Y/X$  and let  $D$  be the event  $R \leq 1/3$ .

a) Determine the value of  $C$ .

b) Are  $X$  and  $Y$  independent? Are they independent conditioned on  $D$ ?

c) Determine the pdf for  $R$  and find  $P(D)$

- 2 Suppose I am interested in constructing an exponential random variable  $Y$  with  $\lambda = 10$ , but Matlab only spits out random variables  $X$  that are Gaussian  $\mathcal{N}(0, 1)$ . Assuming that Matlab can also evaluate the  $Q$ -function ( $Q(c) = \int_c^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ ), develop a procedure to construct  $Y$  from  $X$  so that  $Y$  has the desired target distribution. (*hint: consider taking an intermediate step*).

- 3 Suppose the professional sports playoffs have begun and we have  $m$  games in the first round ( $2m$  teams). Because of new stringent testing procedures, each team has been disqualified due to excessive player steroid use, independently, with probability  $p$ . Let  $N$  be the number of teams still eligible after the disqualifications. Let  $C$  be the number of games that will be played in the first round (i.e., the number of games in which both teams are still eligible). Find  $E[C|N = n]$ . (*hint: remember that the expectation is a linear operator*).

*Extra space if necessary. Please denote which problem you are using this extra space for.*

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