University of Illinois at Urbana-Champaign

ECE 434: Random Processes

Fall 2005
Probability Quiz

Monday, September 12, 2005

Name: _____

- You have one hour for this quiz. The quiz is closed book and closed note.
- Calculators, laptop computers, Palm Pilots, two-way e-mail pagers, etc. may not be used.
- Write your answers in the spaces provided.
- **Please show all of your work. Answers without appropriate justification will receive very little credit.** If you need extra space, use the back of the previous page.

Score:

1. _____ (6 pts.)

2. _____ (12 pts.)

3. _____ (8 pts.)

Total: _____(26 pts.)

Problem 1 *(6 points)* Let X have the pdf $f_X(x) = \begin{cases} \frac{\sin(x)}{2} & x \in [0, \pi] \\ 0 & \text{else} \end{cases}$

(a) Find the cumulative distribution function F_X. In particular, what is $F_X(2\pi)$?

$F_X(2\pi) =$

(b) Compute $E[\sin(X)]$.

Problem 2 *(12 points)* Suppose X and Y are jointly continuous random variables distributed over the unit square with the joint pdf given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{3x^2}{2} + 2xy & x, y \in [0,1] \\ 0 & \text{else} \end{cases}$$

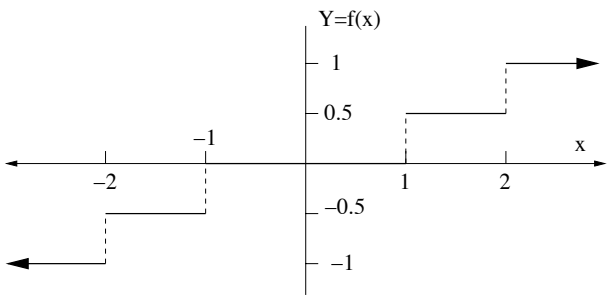
(a) Calculate $E[X]$.

(b) Are X and Y independent? Briefly justify your answer.

(c) Calculate the pdf, $f_Y(y)$, of Y. Be sure to specify it for $-\infty < y < \infty$.

(d) Calculate the conditional density $f_{X|Y}(x|y)$. Be sure to indicate what values of y it is well-defined for, and for such y, specify it for $-\infty < x < \infty$.

Problem 3 *(8 points)* Suppose a $N(0, 9)$ random variable X is passed through the quantizer function f shown. The output is $Y = f(X)$.



(a) Express the pmf of Y in terms of the Q function. (Check your answer. Make sure your answer is positive for the right values of y.)

(b) Express the variance of Y in terms of the Q function. Give as simple an answer as possible.