University of Illinois **Fall 2024**

ECE 515/ME 540: Problem Set 1
State Space models

Due: Wednesday, September 4, 11:59pm
Reading: Course notes, Chapter 1

1. **[State space realizations for a transfer function]**
   Consider the SISO LTI system with the transfer function
   $$G(s) = \frac{s+1}{(s+2)(s+3)(s+4)}.$$
   (a) Obtain a state space model of the system in the controllable canonical form
   (b) Now obtain a state space representation in the observable canonical form.
   (c) Using partial fraction expansion to obtain a representation with a diagonal state matrix A (modal form).

2. **[State space realizations for a transfer function (version 2)]**
   Repeat the previous problem for the SISO LTI system with the transfer function
   $$G(s) = \frac{2s^3+1}{(s+2)(s+3)(s+4)}.$$

3. **[Linearization about an equilibrium]**
   Consider the nonlinear system with the state equation:
   $$\ddot{y} + \dot{y} + y^2 = u^3$$
   (a) Find the linear system obtained by linearizing about the equilibrium $y = u = 1$
   (b) Express the linearized system in state space form. To be definite, give the canonical controllable form of the state space system.

4. **[Pole placement of system in controllable canonical form by state feedback]**
   Consider the following state space model in controllable canonical form:
   $$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 4 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$
   $$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x$$
   (a) What is the characteristic polynomial and what are its roots? (As we will see somewhat later in the course, the characteristic polynomial is the denominator of the transfer function and its roots are the poles of the system.) Hint: One pole is -1.
   (b) Let $u = -Kx + r$ where
   $$K = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}.$$
   We interpret r as the new control input and the system with input r and output y now has state feedback. Give the new state space model with input r and output y. It should again be in controllable canonical form.
   (c) For what choice of K are the roots of the new system $-1, -2, -2$?