Name: ____________________ **NetID: __________**

ECE 515 / ME 540 **Fall 2024 Midterm 2. Nov. 12, 2024**

75 min; 2 page notes 2-sided OK; closed book; no calculators

1. **[Problem 1 (32 points)]**
   Consider the following SISO LTI system:

$$\dot{x} = \begin{bmatrix} 0 & 4 & 5 \\ 2 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x.$$

   (a) Is the system controllable? Justify your answer.

   (b) Is the system observable? Justify your answer.

   (c) Is the system stabilizable? Justify your answer.

   (d) Is the system BIBO stable? Justify your answer.

2. **[Problem 2 (20 points)]**
Consider the following transfer function for a MIMO LTI system.

$$G(s) = \begin{bmatrix} \frac{1}{s+1} + \frac{2}{s+2} + \frac{3}{s+3} & \frac{-2}{s+2} + \frac{6}{s+3} \\ \\ \frac{-2}{s+2} + \frac{1}{s+3} & \frac{1}{s+1} + \frac{2}{s+2} + \frac{2}{s+3} \end{bmatrix}.$$

(a) What is the smallest possible model order for an LTI system that realizes G? Show your work.

(b) Find matrices A, B and C of an LTI system that realizes G with the minimum model order.

3. **[Problem 3 (24 points)]**
Consider the following SISO LTI system:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x.$$

(a) Sketch the two dimensional phase plot/vector field for the ode $\dot{x} = Ax$ for when the control is zero. Identify all the equilibrium points and determine whether they are stable in any of the senses we've considered this semester.

(b) What is the controllable subspace, Σ_c?

(c) What is the unobservable subspace, $\Sigma_{\bar{o}}$?

4. **[Problem 4 (24 points)]**
Suppose the dual of an LTI system with matrices A, B, C, D is equal to itself. That is, $A = -A^*, B = C^*, C = B^*, D = D^*$. (Hint: Think of all properties of dual systems.)

   (a) If the system is controllable must it also be observable? Explain your answer.

   (b) Must the system be stable in any of the senses discussed in class? Explain your answer.

   (c) What can be said about the eigenvalues of A? Explain your answer.