Name: ____________________ **NetID:** __________

ECE 515 / ME 540 Fall 2024 Midterm 1, October 3, 2024

75 min; 1 page notes 2-sided OK; closed book; no caculators; scan & upload to gradescope when done.

1. **[Problem 1 (24 points)]**
   Consider the SISO nonlinear system with state equation $\dot{y} = \sin(2y + u) + e^{-3y+2u} - 1$.

   (a) Find the linear system obtained by linearizing about the equilibrium point $y_e = u_e = 0$.

   (b) If the input u is identically zero, in what sense(s), if any, is the equilibrium point 0 stable for the linearized system? Briefly explain your answer.

   (c) If the input u is identically zero, in what sense(s), if any, is the equilbrium point 0 stable for the original nonlinear system? Briefly explain your answer.

2. **[Problem 2 (20 points)]**

Consider the dynamical system $\dot{x} = \begin{pmatrix} -3 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{pmatrix} x.$

(a) What is the stable subspace Σ_s of the system?

(b) Consider the new system given by the change of coordinates $\overline{x}(t) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix} x(t).$

What is the stable subspace of the new system?

3. **[Problem 3 (16 points)]**

Suppose A, B and C are each 2×2 matrices such that A has eigenvalues 1,2, B has eigenvalues $3, 4$, and C has eigenvalues $5, 6$.

(a) What are the eigenvalues of e^{At} for $t \geq 0$?

(b) What are the eigenvalues of the 4×4 matrix $M = \begin{pmatrix} A & 0_{2\times 2} \\ B & C \end{pmatrix}$?

4. **[Problem 4 (24 points)]**

Let $A = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 5 \end{pmatrix}$.

(a) What is the dimension of the column span of A?

(b) What is the dimension of the row span of A

(c) What is the dimension of the null space of A?

5. **[Problem 5 (16 points)]**
Suppose v^1, v^2, v^3 is a basis for a vector space $\mathcal{V}$ over the field of complex numbers with an inner product. Let G denote the Gramian matrix with i, j^{th} element given by $g_{i,j} = \langle v^i, v^j \rangle$.

   (a) Must G be nonsingular? If yes, explain why. If no, give a counter example.

   (b) Suppose x is a vector in $\mathcal{V}$ and suppose that $\alpha = \begin{pmatrix} \langle v^1, x \rangle \\ \langle v^2, x \rangle \\ \langle v^3, x \rangle \end{pmatrix}$. Let β denote the coordinate vector of x with respect to the basis v^1, v^2, v^3. Derive an expression for β in terms of α and G.