ECE 498YVS-EMC  Homework 2  Due: Wednesday, February 10, 2020, 11:59PM Central Time

- Homeworks are due Wednesday at 11:59 p.m (Champaign local time). Late homework will not be accepted.
- Put name and NetID on the top of every sheet. Scan and submit your homework through Gradescope.
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions.
- One problem is bonus problem for undergraduate student, but it is required if you are a graduate student.
- Penalties for cheating on homework: 0 points on that homework for first offense and an F on the course for any subsequent offense.

Recommended Reading: Paul: Lectures Chapter 3-4

1. Determine one-sided Fourier series expansion coefficients \( c_n, n = 0, 1, 2, \ldots \) for the waveform below. And write the one-sided Fourier series expansion (Compact form). Hint: \( \int x e^{ax} dx = e^{ax}(\frac{x}{a} - \frac{1}{a^2}) \)

\[
\begin{align*}
C_0 &= \int_0^T t \cdot \frac{A}{T} dt = \frac{A}{T^2} \cdot \frac{T^2}{2} = \frac{A}{2} \\
C_n &= \int_0^T t \cdot \frac{A}{T} e^{-jnw_0 t} dt \\
&= \frac{A}{T^2} \int_0^T te^{-jnw_0 t} dt \\
&= \frac{A}{T^2} \left[ \frac{T e^{-jnw_0 T} - \left( \frac{1}{jnw_0} \right) e^{-jnw_0 t}}{jnw_0} \right]_0^T \\
&= \frac{A}{T^2} \left( \frac{T e^{-jnw_0 T}}{jnw_0} - \left( \frac{1}{jnw_0} \right) \right) \\
&= \frac{A}{T^2} \left( \frac{e^{-jnw_0 T}}{jnw_0} - \frac{1}{jnw_0} \right) \\
&= \frac{A}{T^2} \left( \frac{e^{-jnw_0 T}}{jnw_0} - e^{-j2\pi n} \right) \\
&= \frac{A}{T^2} \frac{e^{-j2\pi n T}}{-j2\pi n} = \frac{A}{2\pi n} \cos(2\pi n + \frac{\pi}{2})
\end{align*}
\]

\( \mathcal{F}(\omega) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{A}{2\pi n} \cos(2\pi n + \frac{\pi}{2}) \)
2. Determine the magnitude of the output of the system $|Y(j\omega)|$ shown below at $\omega = 50 \times 10^6$ rad/s.

$|X(j\omega)|_{\omega=50 \times 10^6 \text{ rad/}\text{s}} = 120 \text{ dB} \mu\text{V} - \Delta_1 - \Delta_2 = 78.06 \text{ dB}.$

$|H(j\omega)|_{\omega=50 \times 10^6 \text{ rad/}\text{s}} = -30 \text{ dB} + 20 \log_{10} \left( \frac{50 \times 10^6}{10^5} \right) = -30 \text{ dB} + 53.98 \text{ dB} = 23.98 \text{ dB}.$

$|Y(j\omega)|_{\omega=60 \times 10^6 \text{ rad/}\text{s}} = 78.06 + 23.98 = 102.04 \text{ dB}.$
3. A 5 V, 10 MHz oscillator having a rise/fall time of 10ns and a 50% duty cycle is applied to a gate shown below.

a) Determine the value of the capacitance such that the fifth harmonic is reduced by 20 dB in the gate voltage \( V_G(t) \).

b) Using spectrum bound, estimate the level of this fifth harmonic at the source.

c) (Bonus problem) Calculate the exact value of the fifth harmonic at the source.

\[
\text{a) } 20 \log_{10} \left( \frac{V_{G, \text{with } R} (f_5)}{V_{G, \text{no } R} (f_5)} \right) = -20 \text{ dB}.
\]

\[
\left| \frac{V_{G, \text{with } R} (f_5)}{V_{G, \text{no } R} (f_5)} \right| = \frac{1}{10} = \frac{1}{10}.
\]

\[
V_{G, \text{no } R} = V_S \cdot \frac{10^3}{10^3 + 10^3} = \frac{1}{2} V_S.
\]

\[
V_{G, \text{with } R} = V_S \cdot \frac{R_{II}}{10^3 + R_{II}} = V_S \cdot \frac{10^3}{10^3 + 10^3 + 10^3} \frac{1}{1 + 10^3 j\omega C}
\]

\[
= V_S \cdot \frac{1}{1 + 10^3 j\omega C + l}
\]

\[
\text{So } \left| \frac{2}{2 + 10^3 j\omega C} \right| = 10^{-1}.
\]

\[
\frac{4}{10^6 \omega^2 C^2 + 4} = 0.01.
\]

\[
4 = \frac{a^2}{10^2 \omega^2 C^2} + 0.04.
\]

\[
C = \sqrt{\frac{3.96}{10^4 W_5^2}} = \sqrt{\frac{3.96}{10^2 W_5}} = \frac{1.99}{10^2 \cdot 50 \times 10^6 \Omega} = 63,34 \text{ pF}
\]

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b) \[ T = \frac{1}{f} = \frac{1}{10 \times 10^6} = 0.1 \mu s \]

\[ T = D T = 50\% \times 0.1 \mu s = 0.05 \mu s. \]

\[ \frac{1}{\pi T} = \frac{10^6}{\pi \times 0.05} = 6.37 \text{ MHz}. \]

\[ \frac{1}{\pi T r} = \frac{10^9}{\pi \times 10} = 31.83 \text{ MHz}. \]

Ions rise time \( \rightarrow \) both \( -20 \text{ dB/decade} \ \Delta_1 \)

and \( -40 \text{ dB/decade} \ \Delta_2 \)

\[ \Delta_1 = -20 \log_{10} \left( \frac{\frac{1}{31.83}}{6.37} \right) = -13.97 \text{ dB} \]

\[ \Delta_2 = -40 \log_{10} \left( \frac{\frac{5}{31.83}}{1} \right) = -7.85 \text{ dB} \]

\[ \text{Level}_{f_0} = 20 \log_{10} \left( \frac{\frac{5}{1 \mu V}}{1} \right) - \Delta_1 - \Delta_2 \]

\[ = 133.98 - 13.97 - 7.85 \text{ dBuV} \]

\[ = 112.16 \text{ dBuV}. \]

c) \[ |C_5| = 2AD \left( \frac{\sin \left( \frac{n \omega_0 T}{2} \right)}{\frac{n \omega_0 T}{2}} \right) - \frac{\sin \left( \frac{n \omega_0 T r}{2} \right)}{\frac{n \omega_0 T r}{2}} \]

\[ = 5 \left( \frac{\sin \left( 5 \times 2\pi \times 10^7 \times 0.5 \times 10^{-9} / 2 \right)}{5 \times 2\pi \times 10^7 \times 0.5 \times 10^{-9} / 2} \right) - \frac{\sin \left( 5 \times 2\pi \times 10^7 \times 10 \times 0.9 / 2 \right)}{5 \times 2\pi \times 10^7 \times 10 \times 0.9 / 2} \]

\[ = 5 \cdot \frac{\sin (2.5 \pi)}{2.5 \pi} \cdot \frac{\sin (0.5 \pi)}{0.5 \pi} \]

\[ = 5 \cdot \frac{1}{7.854} \cdot \frac{1}{1.5708} \]

\[ = 0.40528 \text{ V} \]

\[ = 112.155 \text{ dBuV}. \]
4. A typical coaxial cable is RG6U, which has an interior 18-gauge (radius 20.15 mils) solid wire, an interior shield radius of 90 mils, and an inner insulation of foamed polyethylene having a relative permittivity of 1.45. Determine the per-unit-length capacitance, inductance, and the velocity of propagation relative to that of free space.

\[
\begin{align*}
 r_1 &= 20.15 \text{ mils}, \quad r_2 = 90 \text{ mils}, \quad \varepsilon_r = 1.45 \text{ F/m}, \quad \mu_0 = 4\pi \times 10^{-7} \text{ H/m}. \\
\varepsilon_0 &= 8.854 \times 10^{-12} \text{ F/m}.
\end{align*}
\]

\[
\ell = \frac{\mu_0 \varepsilon_r}{2\pi} \ln \left( \frac{r_2}{r_1} \right) = \frac{4\pi \times 10^{-7}}{2\pi} \ln \left( \frac{90}{20.15} \right) = 0.299 \text{ mF/m}.
\]

\[
C = \frac{2\pi \varepsilon_0 \varepsilon_r}{\ln \left( \frac{r_2}{r_1} \right)} = \frac{2\pi \times 8.854 \times 10^{-12} \times 1.45}{\ln \left( \frac{90}{20.15} \right)} = 53.9 \text{ pF/m}.
\]

\[
V = \frac{C_0}{\sqrt{\varepsilon_r}} = 0.83 C
\]

5. A microstrip line is constructed on a FR-4 board having a relative permittivity of 4.7. The board thickness is 64 mils and the land width is 10 mils. Determine the per-unit-length capacitance and inductance as well as the effective relative permittivity.

\[
\begin{align*}
\varepsilon_r' &= \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \frac{1}{\sqrt{1 + 10h/w}} \\
&= 2.85 + \frac{3.7}{2} \frac{1}{\sqrt{1 + 10 \times 6.4}} \\
&= 3.079 \text{ F/m}
\end{align*}
\]

\[
V_p = \frac{C}{\sqrt{\varepsilon_r'}} = 0.57 C
\]

\[
\begin{align*}
\ell &= \frac{\mu_0 \varepsilon_0 \varepsilon_r'}{C_0} = \frac{\varepsilon_r'}{C_0^2 C} = \frac{3.079}{(3 \times 10^8)^2 \times 43.45 \times 10^{-12}} \\
&= 787.36 \text{ nH/m}.
\end{align*}
\]

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