ECE 498YVS-EMC   Homework 3   Due: Thursday, September 19, 2024, 11:59PM Central Time

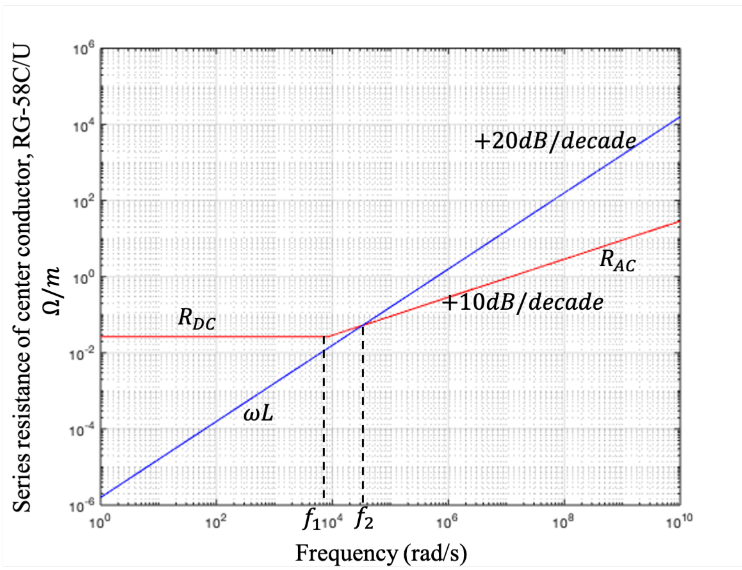
Recommended Reading: Paul: Chapter 4.

1. The skin effect governs the behavior of all conductors. As an example, Figure below depicts the series resistance of RG-58C/U coaxial cable plotted as a function of frequency. The plot uses log-log axes. Assume the center conductor radius $r = 0.455$ mm, and per-unit-length inductance $L = 253$ nH/m. The conductivity of copper is $\sigma = 5.8 \times 10^7$ S/m.

   a) Calculate the per-unit-length DC resistance of center conductor.

   b) Calculate frequency f_1 where skin depth approximately take effect on center conductor ($R_{AC} > R_{DC}$).

   c) Calculate frequency f_2 where $\omega L > R_{copper}$.



Solution:
a) per-unit-length DC resistance is

$$R_{DC} = 1/\sigma A = \frac{1}{\sigma \pi r^2} = \frac{1}{(5.8 \times 10^7)\pi(0.455 \times 10^{-3})^2} = 0.0265\ \Omega$$

b) At high frequency, the skin depth is $\delta = \sqrt{\frac{1}{\pi f \mu \sigma}}$, per-unit-length AC resistance is

$$R_{AC} = \frac{1}{\sigma 2\pi r \delta}$$

we have $R_{AC} > R_{DC}$, which is

$$R_{AC} = \frac{1}{\sigma 2\pi r \delta} > R_{DC} = \frac{1}{\sigma \pi r^2}$$

Or,

$$r > 2\delta = 2\sqrt{\frac{1}{\pi f \mu \sigma}}$$

$$f_1 = \frac{4}{r^2 \pi \mu \sigma} = 84.38\ kHz$$

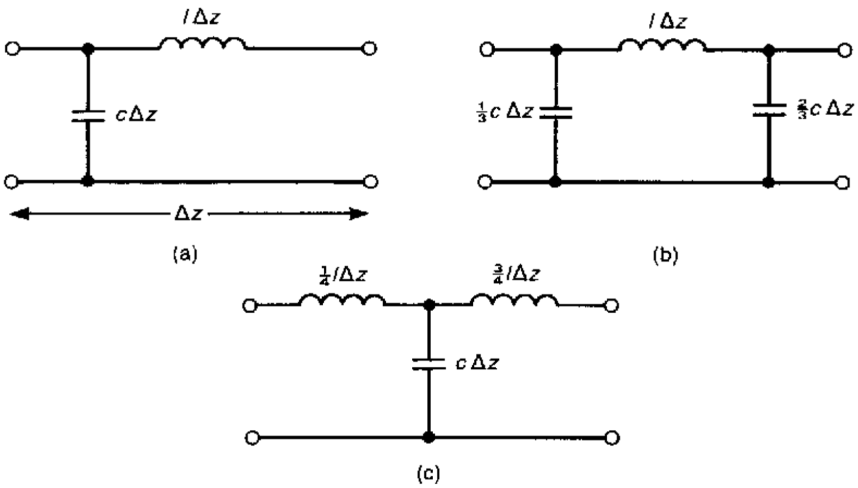
c) Since at high frequency $2\pi f_2 L > R_{AC}$, we have

$$2\pi f_2 L > \frac{1}{\sigma 2\pi r\delta} = \frac{\sqrt{\pi f\mu\sigma}}{\sigma 2\pi r}$$

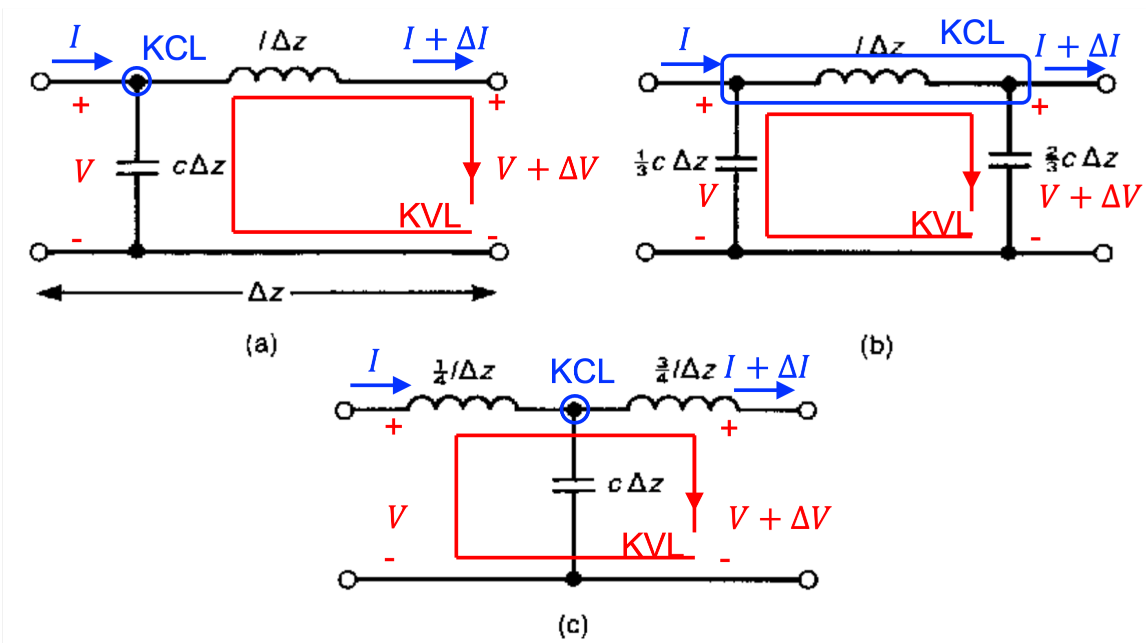
$$f_2 = \frac{\pi\mu}{\sigma(4\pi^2 rL)^2} = 3.296\ kHz$$

Note: You may notice $f_2 < f_1$ for this example, that's because the lines in the figure are not drawn to scale.

2. For the per-unit-length representations of a lossless transmission line shown below in three structures, derive the transmission-line equations in the limit as $\Delta z \to 0$. Note that the total per-unit-length inductance and capacitance in each circuit are l and c, respectively. This shows that the structure of the per-unit-length equivalent circuit is not important in the limit as $\Delta z \to 0$.



Solution:



a) Using KVL: $V - l\triangle z\frac{dI}{dz} = V + \triangle V \quad \Longrightarrow \quad \frac{dV}{dz} = -l\frac{dI}{dt}$
Using KCL: $I = c\triangle z\frac{dV}{dt} + I + \triangle I \quad \Longrightarrow \quad \frac{dI}{dz} = -c\frac{dV}{dt}$
The above two equations are the Telegrapher's equation, from which we can also derive the wave equation

$$\frac{d^2V}{dz^2} = lc\frac{d^2V}{dt^2}$$

b) Using KVL: $V - l\triangle z\frac{dI}{dz} = V + \triangle V \quad \Longrightarrow \quad \frac{dV}{dz} = -l\frac{dI}{dt}$
Using KCL: $I = \frac{1}{3}c\triangle z\frac{dV}{dt} + \frac{2}{3}c\triangle z\frac{dV}{dt} + I + \triangle I \quad \Longrightarrow \quad \frac{dI}{dz} = -c\frac{dV}{dt}$

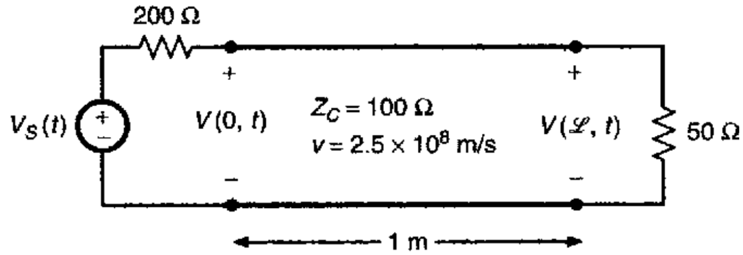
And the wave equation is

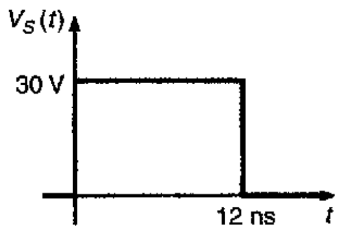
$$\frac{d^2V}{dz^2} = lc\frac{d^2V}{dt^2}$$

c) Using KVL: $V - \frac{1}{4}l\triangle z\frac{dI}{dz} - \frac{3}{4}l\triangle z\frac{dI}{dz} = V + \triangle V \quad \implies \quad \frac{dV}{dz} = -l\frac{dI}{dt}$
Using KCL: $I = c\triangle z\frac{dV}{dt} + I + \triangle I \quad \implies \quad \frac{dI}{dz} = -c\frac{dV}{dt}$
And the wave equation is

$$\frac{d^2V}{dz^2} = lc\frac{d^2V}{dt^2}$$

3. Sketch the input voltage to the line V(0, t) and the load voltage V(L, t) for the problem depicted in the figure below for $0 < t < 20$ ns. What should these plots converge to in the steady state?





Solution:

The characteristic impedance of the coaxial cable is $Z_c = \sqrt{\frac{l}{c}} = \sqrt{\frac{0.25\times10^{-6}}{100\times10^{-12}}} = 50\ \Omega$

The propagation speed in the coax is $v_p = \frac{1}{\sqrt{lc}} = \frac{1}{\sqrt{0.25\times10^{-6}\times100\times10^{-12}}} = 2\times10^8\ m/s$

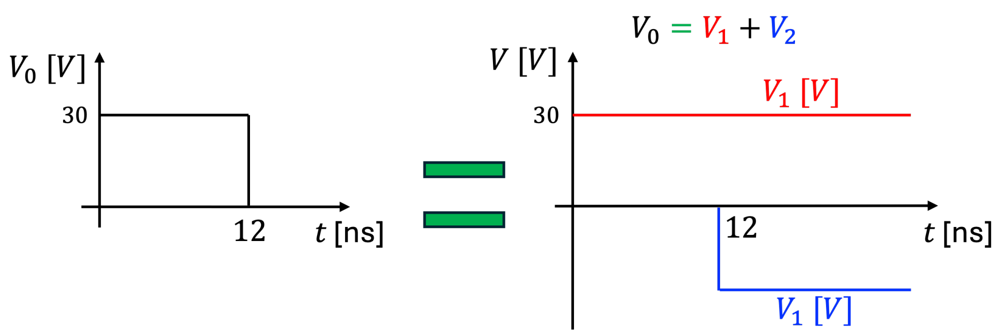
The delay time of the coaxial cable is $T_d = L/v_p = \frac{400}{2\times10^8} = 2\ \mu s$

For the transmission line, the injection coefficient is $k = \frac{Z_c}{R_s+Z_c} = \frac{50}{150+50} = \frac{1}{4}$

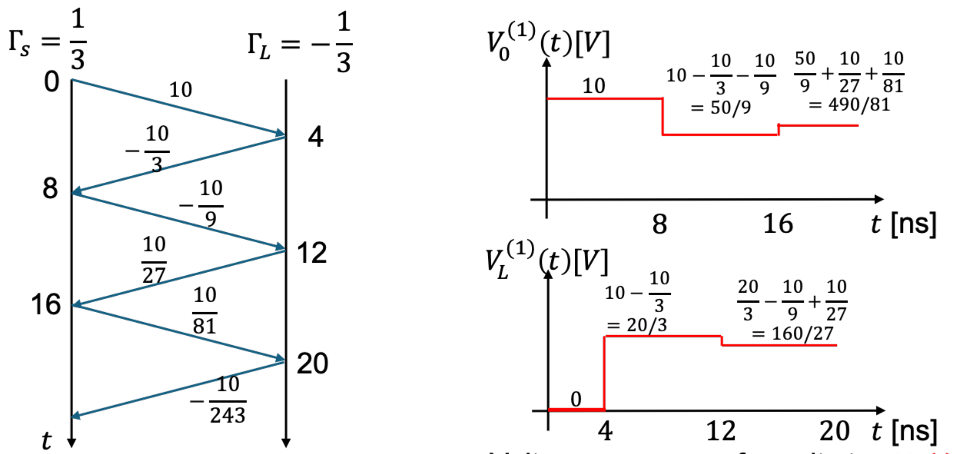
Reflection coefficient at the source side is $\Gamma_s = \frac{R_s-Z_c}{R_s+Z_c} = \frac{150-50}{150+50} = \frac{1}{2}$

Reflection coefficient at the load side is $\Gamma_L = \frac{R_L-Z_c}{R_L+Z_c} = \frac{0-50}{0+50} = -1$

We notice that the input voltage $V_0 = V_1 + V_2$

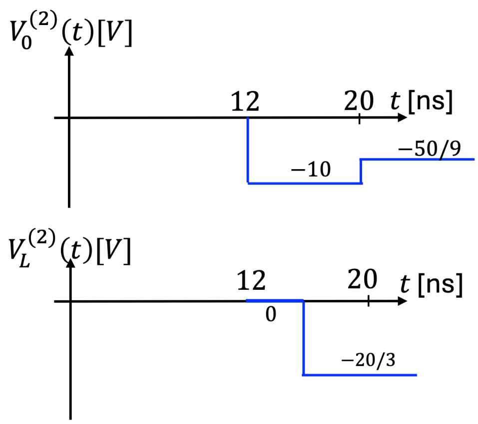


We first draw the bounce diagram for input voltage as $V_1(t) = 30u(t)$



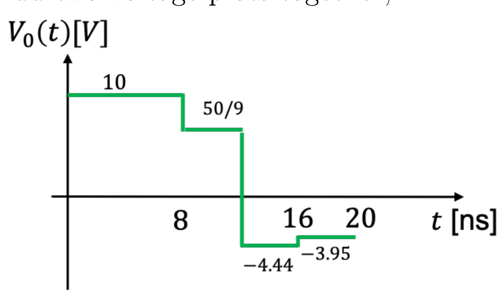
Voltage responses for unit step $V_1(t) = 30u(t)$

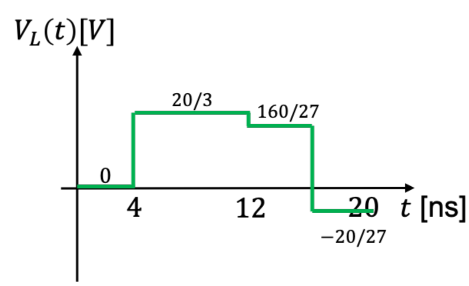
Next, we plot the voltage for $V_2(t) = -30u(t-12)$



Voltage responses for unit step $V_2(t) = -30u(t-12)$

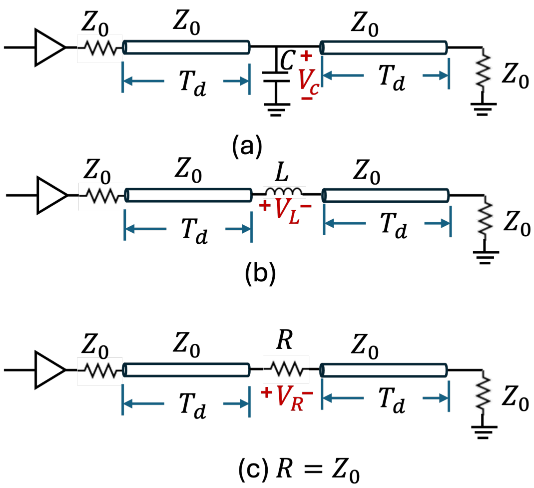
Add two voltage plots together,



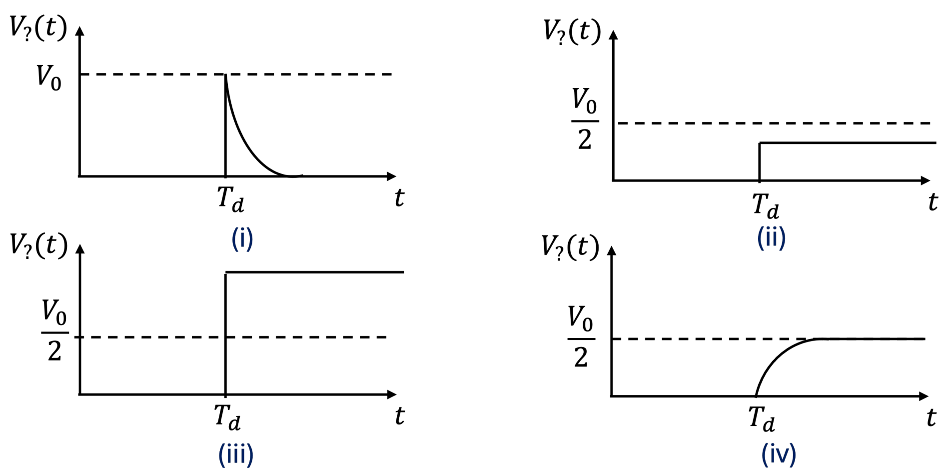


The steady state voltage $V(0,\infty) = V(L,\infty) = 0\ V$ since the source voltage is 0V when $t \to \infty$.

4. In transmission lines, various types of discontinuities can occur, such as reactive (capacitive or inductive) and resistive mismatches. These discontinuities can affect the voltage $V(z)$ along the transmission line.
Consider a constant voltage $V_{in}(t) = V_0$ is injected into four transmission lines with different discontinuities (parallel C, series L, and series R where $R = Z_0$). The line is matched everywhere else (source impedance, characteristic impedance, and load impedance are all Z_0.



a) The four voltage plots are related to the voltage at the discontinuity immediately after the signal reaches that point. Match circuit diagram (a) to (c) to voltage output (i) to (iv). Hint: one voltage output can't be matched.



b) Explain in part (a), why voltage in (i) is V_0, while in figure (iv) is only $V_0/2$.

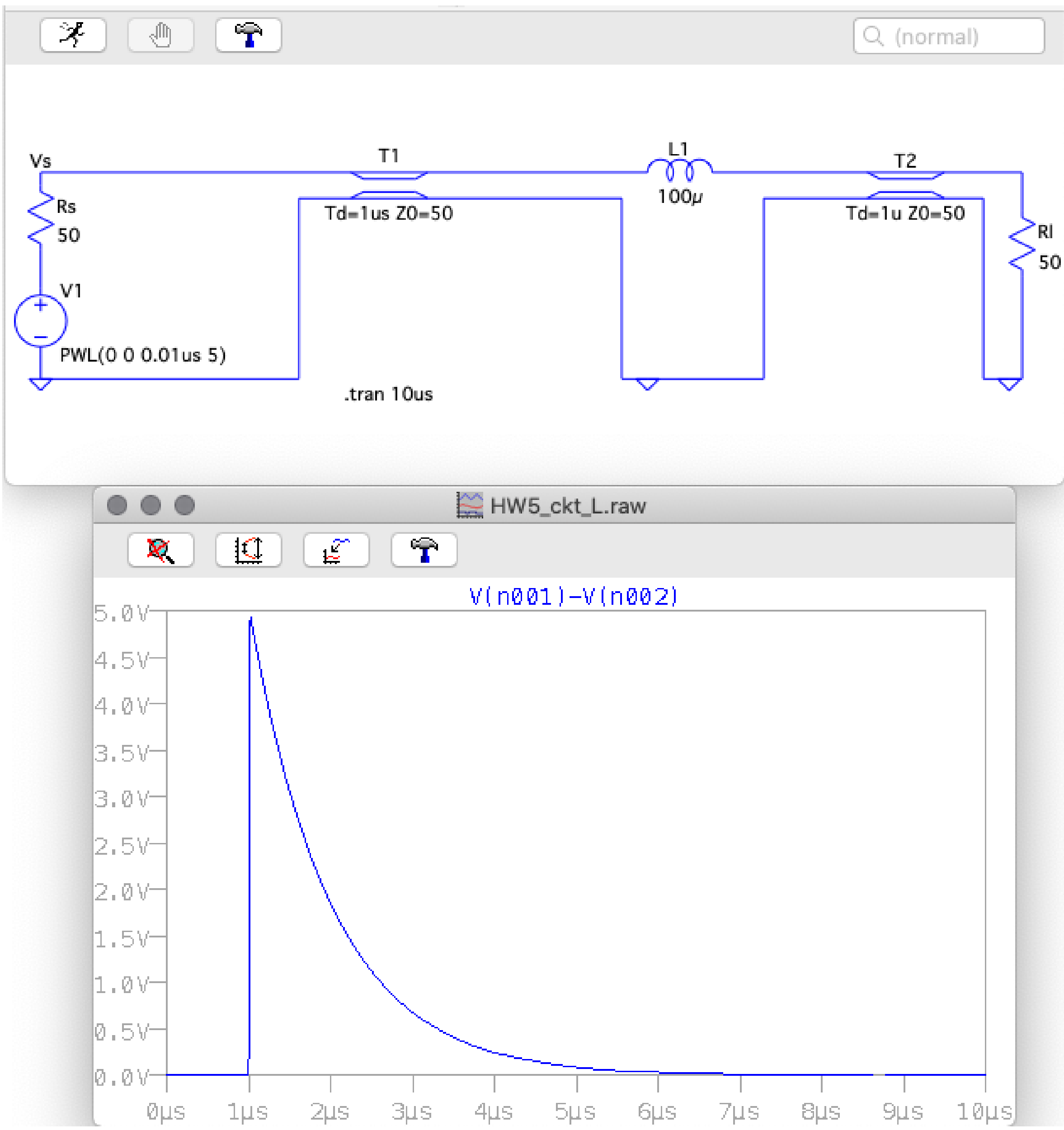
c) Discuss how reactive elements (capacitor and inductor) impact transient behavior and reflection differently compared to the resistive mismatches.

d) How do reactive (L and C) discontinuities impact on Signal Integrity and radiated emissions?

Solution: a) circuit (a) is with voltage (iv), circuit (b) is with voltage (i), and circuit (c) is with voltage (ii).

b) The injection coefficient on the source side $\tau = \frac{Z_0}{Z_0+Z_0} = 1/2$, so the forward going voltage into the transmission line is $V_{in} = \tau V_0 = V_0/2$.

i) For circuit a) with parallel capacitor, when the forwarding wave reaches the capacitor, capacitor starts to store energy with 0 V across it. When charge accumulates on the capacitor, V_c will increase exponentially $V_c(t) = \frac{V_0}{2}(1 - e^{-\frac{t}{Z_0 C}})$. When $t \to \infty$, parallel capacitor behaves like an open circuit, which results a perfect impedance match along transmission line, so the system reaches a steady state and the voltage will stay at $V_0/2$.
ii) For circuit b) with series inductor, when the forwarding wave reaches the series inductor, the inductor resists the change in current $V_L = L\frac{dI}{dt}$ (act like an open circuit). So the reflection coefficient becomes $\Gamma = \frac{\infty - Z_0}{\infty + Z_0} = 1$. The forwarding wave and backward going wave both have magnitude of $V_0/2$, which adds up and creates a voltage spike of V_0.

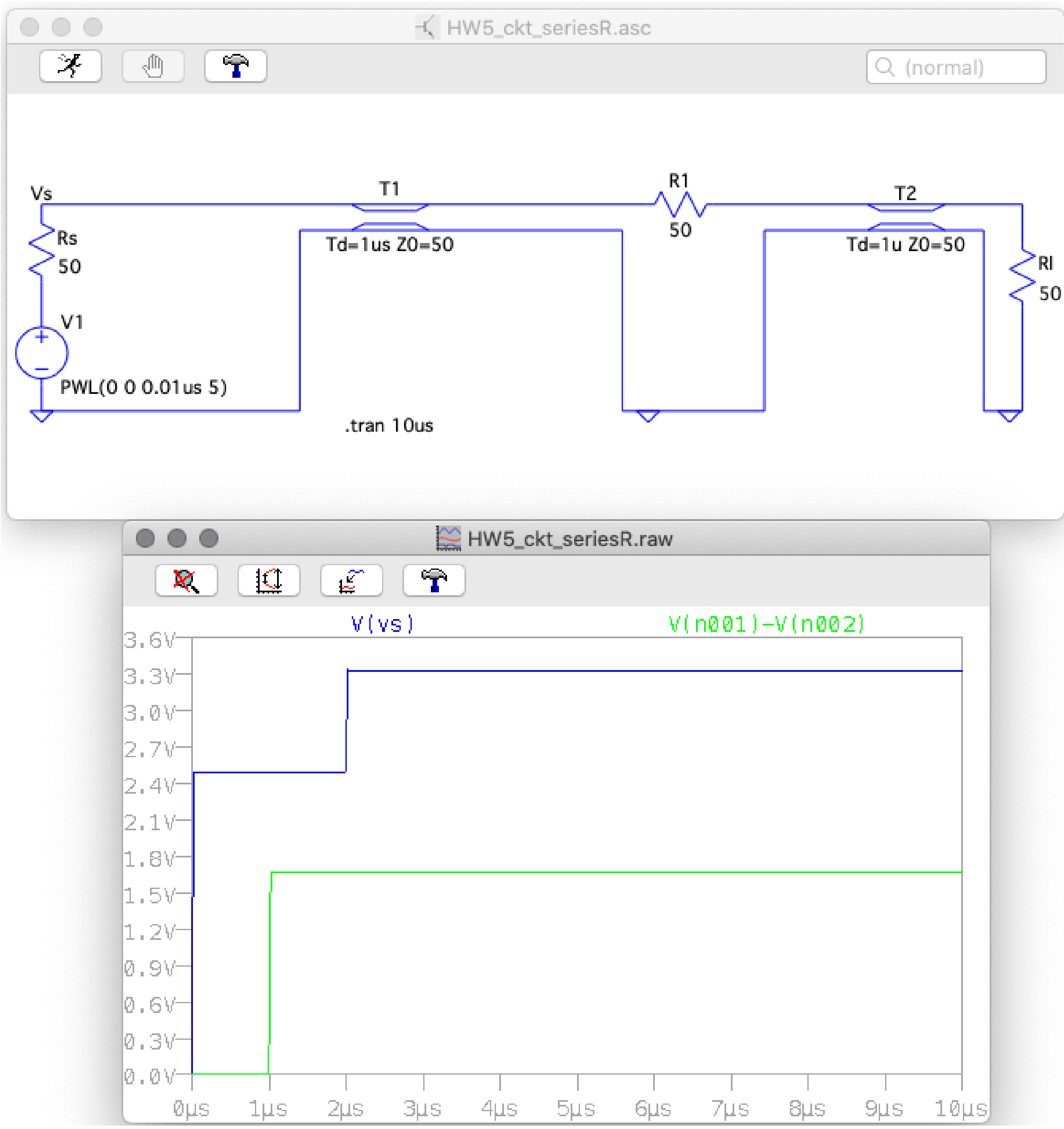


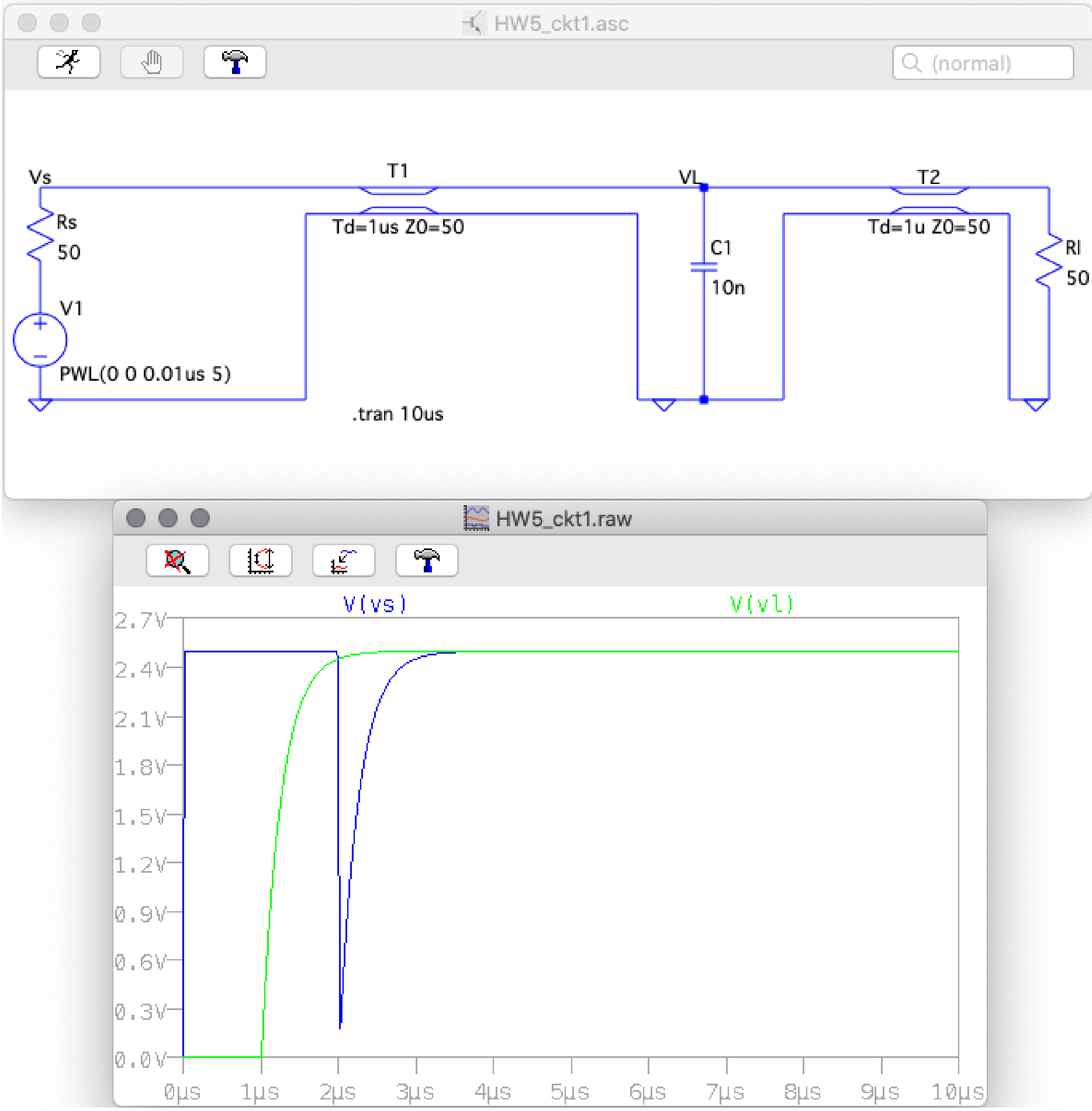
c) Capacitors resist changes in voltage, leading to a delay in voltage buildup across the capacitor during a transient event. Inductors resist changes in current, causing an initial voltage

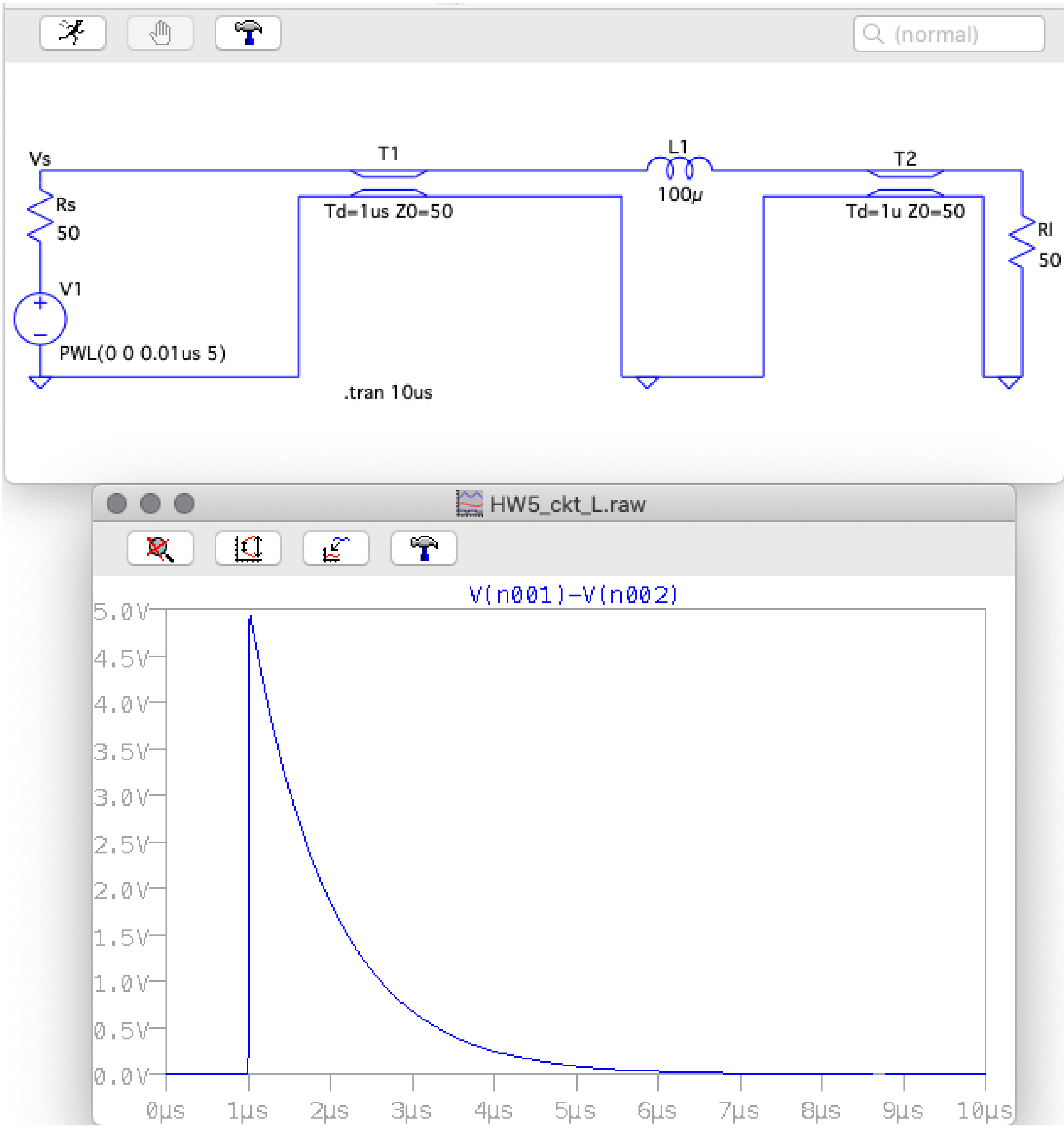
spike when a transient signal first encounters the inductor. The voltage responses for L and C both are exponential functions and with time constant of RC and L/R.
For resistive discontinuity, reflections are not frequency-dependent; they depend purely on the mismatch between the resistor value and the characteristic impedance.

d) Impact on Signal Integrity: The impedance of capacitors and inductors depends on frequency ($Z_c = \frac{1}{j\omega C}$ and $Z_L = j\omega L$). If the input signals has various frequency components, the reflection coefficients are also depending on frequency. The reflection caused by the reactive and resistive elements can create signal distortion, leading to overshoot, undershoot, and ringing.
Impact on radiated emission: At high frequencies, the capacitor in parallel presents a low impedance ($Z_c = \frac{1}{j\omega C}$), which means it can sink or divert high-frequency currents. This action can attenuate high-frequency components, reducing radiated emission.
At high frequencies, inductor impedance ($Z_L = j\omega L$) becomes large, which means that the inductor in series effectively acts as a high impedance or "block" to high-frequency signals, reducing radiated emission.

5. (For Graduate students only) Use LTSPICE to confirm your answer for circuit (a) to (c) in problem 4. You can explore various L, C, and R values in your simulation.




HW5_ckt1.asc
(normal)
Vs
T1
VL
T2
Rs
50
Td=1us Z0=50
C1
10n
Td=1u Z0=50
Rl
50
V1
PWL(0 0 0.01us 5)
.tran 10us
HW5_ckt1.raw
V(vs)
V(vl)
2.7V
2.4V
2.1V
1.8V
1.5V
1.2V
0.9V
0.6V
0.3V
0.0V
0µs
1µs
2µs
3µs
4µs
5µs
6µs
7µs
8µs
9µs
10µs


(normal)
Vs
T1
L1
T2
Rs
50
Td=1us Z0=50
100μ
Td=1u Z0=50
Rl
50
V1
PWL(0 0 0.01us 5)
.tran 10us
HW5_ckt_L.raw
V(n001)-V(n002)
5.0V
4.5V
4.0V
3.5V
3.0V
2.5V
2.0V
1.5V
1.0V
0.5V
0.0V
0µs
1µs
2µs
3µs
4µs
5µs
6µs
7µs
8µs
9µs
10µs