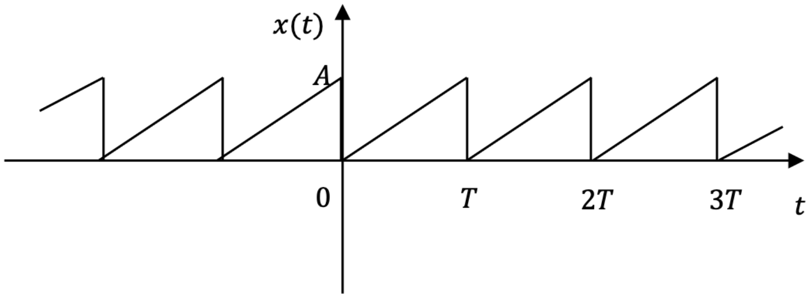
ECE 498YVS-EMC   Homework 2   Due: Thursday, September 12, 2024, 11:59PM Central Time

Recommended Reading: Paul: Lectures Chapter 3-4

1. Determine one-sided Fourier series expansion coefficients (c_n, $n = 0, 1, 2, ...$) for the periodic waveform below. Also write down the one-sided Fourier series expansion (Compact form).
   Hint: $\int xe^{ax}dx = e^{ax}(\frac{x}{a} - \frac{1}{a^2})$



Solution:
The one-sided Fourier series expansion coefficients are

$$c_0 = \frac{1}{T}\int_0^T t \cdot \frac{A}{T}dt = \frac{A}{T^2} \cdot \frac{t^2}{2} \Big|_0^T = \boxed{\frac{A}{2}}$$

$$c_n = \frac{1}{T}\int_0^T t \cdot \frac{A}{T}e^{-jn\omega_0 t}dt = \frac{A}{T^2}\int_0^T te^{-jn\omega_0 t}dt$$

$$= \frac{A}{T^2}e^{-jn\omega_0 t}\left[\frac{t}{-jn\omega_0} - \frac{1}{(-jn\omega_0)^2}\right] \Big|_0^T = \frac{A}{T^2}e^{-jn\omega_0 T} \cdot \frac{T}{-jn\omega_0}$$

$$= j\frac{A}{2\pi n} = \boxed{\frac{A}{2\pi n}\angle 90^\circ}$$

where

$$e^{-jn\omega_0 T} = e^{-j2n\pi} = 1$$

The one-sided Fourier series expansion is

$$\boxed{F(\omega) = \frac{A}{2} + \sum_{n=1}^{\infty}\frac{A}{n\pi}cos(n\omega_0 t + \frac{\pi}{2})}$$

2. A trapezoidal waveform with a peak magnitude of 5V is operating at 10 MHz, with a rise time and fall time of 5 ns and a duty cycle of 50%.

   a) Derive the one-sided Compact form Fourier series representation of the waveform.

   b) Calculate the magnitude of the first five harmonics (C_1 to C_5). Discuss how the harmonics contribute to the overall spectrum AND how this relates to potential EMI concerns.

   c) Determine the magnitude of the first five harmonics if the rise/fall time is changed to 10 ns. Discuss how the change in rise/fall time affects the overall spectrum AND how this relates to potential EMI concerns.

   d) Determine the magnitude of the first five harmonics if the duty cycle is reduced to 40%. Discuss how the change in duty cycle affects the overall spectrum AND how this relates to potential EMI concerns.

   e) Determine the magnitude of the first five harmonics if the waveform operates at 20 MHz instead. Discuss how the change in repetition rate affects the overall spectrum AND how this relates to potential EMI concerns.

Solution:

a) One-Sided Compact Fourier Series of trapezoidal wave form is

$$x(t) = \frac{A\tau}{T} + \frac{2A\tau}{T}\sum_{n=1}^{\infty}\left|\frac{sin(\pi n f_0 \tau)}{\pi n f_0 \tau}\right|\left|\frac{sin(\pi n f_0 \tau_r)}{\pi n f_0 \tau_r}\right| cos(2\pi n f_0 t - \pi n f_o(\tau - \tau_r))$$

Given magnitude $A = 5\ V$, Duty cycle $D = \frac{\tau}{T} = 50\%$, rise time $\tau_r = 5ns$, and period $T = 1/f_0 = 100ns$, pulse width $\tau = DT = 50ns$, the wave is

$$x(t) = 2.5 + 5\sum_{n=1}^{\infty}\left|\frac{sin(\pi n/2)}{\pi n/2}\right|\left|\frac{sin(\pi n \cdot 0.05)}{\pi n \cdot 0.05}\right| cos(2\pi n \cdot 10^7 \cdot t - \pi n \cdot 0.45)$$

b) First Five Harmonics (C_1 to C_5)
The magnitude of the harmonics C_n is given by

$$C_n = \frac{2A\tau}{T}\left|\frac{sin(\pi n f_0 \tau)}{\pi n f_0 \tau}\right|\left|\frac{sin(\pi n f_0 \tau_r)}{\pi n f_0 \tau_r}\right| = 5\left|\frac{sin(\pi n/2)}{\pi n/2}\right|\left|\frac{sin(\pi n \cdot 0.05)}{\pi n \cdot 0.05}\right|$$

First harmonic $C_1 = 5\left|\frac{sin(\pi \cdot 1/2)}{\pi \cdot 1/2}\right|\left|\frac{sin(\pi \cdot 1 \cdot 0.05)}{\pi \cdot 1 \cdot 0.05}\right| \approx 5\left|\frac{2}{\pi}\right|\left|\frac{\pi \cdot 1 \cdot 0.05}{\pi \cdot 1 \cdot 0.05}\right| = \frac{10}{\pi} \approx 3.17\ V$

2nd harmonic $C_2 = 5\left|\frac{sin(\pi \cdot 2/2)}{\pi \cdot 2/2}\right|\left|\frac{sin(\pi \cdot 2 \cdot 0.05)}{\pi \cdot 2 \cdot 0.05}\right| = 0\ V$

3rd harmonic $C_3 = 5\left|\frac{sin(\pi \cdot 3/2)}{\pi \cdot 3/2}\right|\left|\frac{sin(\pi \cdot 3 \cdot 0.05)}{\pi \cdot 3 \cdot 0.05}\right| \approx 1.022\ V$

4th harmonic $C_4 = 5\left|\frac{sin(\pi \cdot 4/2)}{\pi \cdot 4/2}\right|\left|\frac{sin(\pi \cdot 4 \cdot 0.05)}{\pi \cdot 4 \cdot 0.05}\right| = 0\ V$

First harmonic $C_5 = 5\left|\frac{sin(\pi \cdot 5/2)}{\pi \cdot 5/2}\right|\left|\frac{sin(\pi \cdot 5 \cdot 0.05)}{\pi \cdot 5 \cdot 0.05}\right| \approx 0.573\ V$

Harmonic distribution: The first harmonic contains most of the signal's energy. Higher harmonics contribute less. No even harmonics because of duty cycle D=50%.

EMI concern: Higher harmonics can radiate more easily because the wire/trace will become half wave antenna or quarter wave monopole antenna. Proper EMI mitigation techniques may be needed.

c) Calculate first Five Harmonics (C_1 to C_5) when rise/fall time increase to 10 ns
The magnitude of the harmonics C_n is given by

$$C_n = \frac{2A\tau}{T}\left|\frac{sin(\pi n f_0 \tau)}{\pi n f_0 \tau}\right|\left|\frac{sin(\pi n f_0 \tau_r)}{\pi n f_0 \tau_r}\right| = 5\left|\frac{sin(\pi n/2)}{\pi n/2}\right|\left|\frac{sin(\pi n \cdot 0.1)}{\pi n \cdot 0.1}\right|$$

First harmonic $C_1 = 5\left|\frac{sin(\pi \cdot 1/2)}{\pi \cdot 1/2}\right|\left|\frac{sin(\pi \cdot 1 \cdot 0.1)}{\pi \cdot 1 \cdot 0.1}\right| \approx 3.13\ V$

2nd harmonic $C_2 = 5\left|\frac{sin(\pi \cdot 2/2)}{\pi \cdot 2/2}\right|\left|\frac{sin(\pi \cdot 2 \cdot 0.1)}{\pi \cdot 2 \cdot 0.1}\right| = 0\ V$

3rd harmonic $C_3 = 5\left|\frac{sin(\pi \cdot 3/2)}{\pi \cdot 3/2}\right|\left|\frac{sin(\pi \cdot 3 \cdot 0.1)}{\pi \cdot 3 \cdot 0.1}\right| \approx 0.91\ V$

4th harmonic $C_4 = 5\left|\frac{sin(\pi \cdot 4/2)}{\pi \cdot 4/2}\right|\left|\frac{sin(\pi \cdot 4 \cdot 0.1)}{\pi \cdot 4 \cdot 0.1}\right| = 0\ V$

First harmonic $C_5 = 5\left|\frac{sin(\pi \cdot 5/2)}{\pi \cdot 5/2}\right|\left|\frac{sin(\pi \cdot 5 \cdot 0.1)}{\pi \cdot 5 \cdot 0.1}\right| \approx 0.405\ V$

Harmonic distribution: The magnitudes of higher order harmonic are smaller when increasing rise/fall time. The first harmonic are not affected too much.

EMI concern: Increasing the rise/fall time reduces the bandwidth and decreases the likelihood of EMI issues.

d) Calculate first Five Harmonics (C_1 to C_5) when duty cycle reduces to 40%
The magnitude of the harmonics C_n is given by

$$C_n = \frac{2A\tau}{T}\left|\frac{sin(\pi n f_0 \tau)}{\pi n f_0 \tau}\right|\left|\frac{sin(\pi n f_0 \tau_r)}{\pi n f_0 \tau_r}\right| = 5\left|\frac{sin(\pi n \cdot 0.4)}{\pi n \cdot 0.4}\right|\left|\frac{sin(\pi n \cdot 0.05)}{\pi n \cdot 0.1}\right|$$

First harmonic $C_1 = 5\left|\frac{sin(\pi \cdot 1 \cdot 0.4)}{\pi \cdot 1 \cdot 0.4}\right|\left|\frac{sin(\pi \cdot 1 \cdot 0.05)}{\pi \cdot 1 \cdot 0.05}\right| \approx 3.77\ V$

2nd harmonic $C_2 = 5\left|\frac{sin(\pi \cdot 2 \cdot 0.4)}{\pi \cdot 2 \cdot 0.4}\right|\left|\frac{sin(\pi \cdot 2 \cdot 0.05)}{\pi \cdot 2 \cdot 0.05}\right| = 1.15\ V$

3rd harmonic $C_3 = 5\left|\frac{sin(\pi \cdot 3 \cdot 0.4)}{\pi \cdot 3 \cdot 0.4}\right|\left|\frac{sin(\pi \cdot 3 \cdot 0.05)}{\pi \cdot 3 \cdot 0.05}\right| \approx 0.75\ V$

4th harmonic $C_4 = 5\left|\frac{sin(\pi \cdot 4 \cdot 0.4)}{\pi \cdot 4 \cdot 0.4}\right|\left|\frac{sin(\pi \cdot 4 \cdot 0.05)}{\pi \cdot 4 \cdot 0.05}\right| = 0.885\ V$

First harmonic $C_5 = 5\left|\frac{sin(\pi \cdot 5 \cdot 0.4)}{\pi \cdot 5 \cdot 0.4}\right|\left|\frac{sin(\pi \cdot 5 \cdot 0.05)}{\pi \cdot 5 \cdot 0.05}\right| = 0\ V$

Harmonic distribution: Even harmonics are no longer zero.

EMI concern: Change in the duty cycle to something other than 50% means the even harmonics are no longer 0. Even if when duty cycle deviates from 50%, say 50.1%, the even harmonic spectrum magnitude is no longer 0. The magnitude varies when n increase, which could cause potential EMI problem.

e) Calculate first Five Harmonics (C_1 to C_5) when fundamental frequency increases to 20 MHz
The magnitude of the harmonics C_n is given by

$$C_n = \frac{2A\tau}{T}\left|\frac{sin(\pi n f_0 \tau)}{\pi n f_0 \tau}\right|\left|\frac{sin(\pi n f_0 \tau_r)}{\pi n f_0 \tau_r}\right| = 5\left|\frac{sin(\pi n/2)}{\pi n/2}\right|\left|\frac{sin(\pi n \cdot 0.1)}{\pi n \cdot 0.1}\right|$$

First harmonic $C_1 = 5\left|\frac{sin(\pi \cdot 1/2)}{\pi \cdot 1/2}\right|\left|\frac{sin(\pi \cdot 1 \cdot 0.1)}{\pi \cdot 1 \cdot 0.1}\right| \approx 3.13\ V$ @ 20MHz

2nd harmonic $C_2 = 5\left|\frac{sin(\pi \cdot 2/2)}{\pi \cdot 2/2}\right|\left|\frac{sin(\pi \cdot 2 \cdot 0.1)}{\pi \cdot 2 \cdot 0.1}\right| = 0\ V$ @ 40MHz

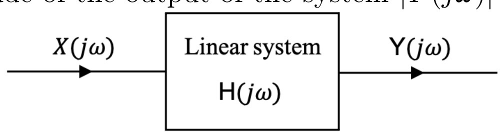
3rd harmonic $C_3 = 5\left|\frac{sin(\pi \cdot 3/2)}{\pi \cdot 3/2}\right|\left|\frac{sin(\pi \cdot 3 \cdot 0.1)}{\pi \cdot 3 \cdot 0.1}\right| \approx 0.91\ V$ @ 60MHz

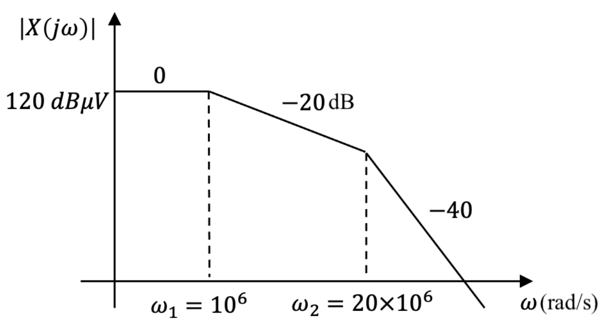
4th harmonic $C_4 = 5\left|\frac{sin(\pi \cdot 4/2)}{\pi \cdot 4/2}\right|\left|\frac{sin(\pi \cdot 4 \cdot 0.1)}{\pi \cdot 4 \cdot 0.1}\right| = 0\ V$ @ 80MHz

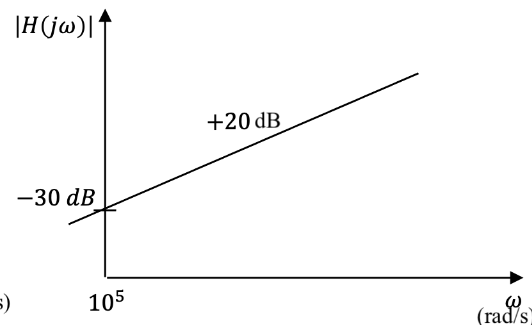
First harmonic $C_5 = 5\left|\frac{sin(\pi \cdot 5/2)}{\pi \cdot 5/2}\right|\left|\frac{sin(\pi \cdot 5 \cdot 0.1)}{\pi \cdot 5 \cdot 0.1}\right| \approx 0.405\ V$ @ 100MHz

Harmonic distribution: Although the magnitude for the first five harmonics are the same as in part (c), but now they are located at higher frequencies.
EMI concern: As the fundamental frequency increases, the harmonics become more widely spaced. This higher frequency content can lead to increased potential for interference with other electronic devices.

3. Determine the magnitude of the output of the system $|Y(j\omega)|$ shown below at $\omega = 50 \times 10^6$ rad/s.







Solution:

$$\triangle_1 = -20log_{10}\left(\frac{\omega_2}{\omega_1}\right) = -20log_{10}\left(\frac{20 \times 10^6}{1 \times 10^6}\right) = -26.02\ dB$$

$$\triangle_2 = -40log_{10}\left(\frac{\omega}{\omega_2}\right) = -40log_{10}\left(\frac{50 \times 10^6}{20 \times 10^6}\right) = -15.92\ dB$$

$$\mid X(j\omega) \mid_{\omega=50\times10^6 rad/s} = 120\ dB\mu V - \triangle_1 - \triangle_2 = 78.06\ dB\mu V$$

$$\mid H(j\omega) \mid_{\omega=50\times10^6 rad/s} = -30\ dB\mu V + 20log_{10}\left(\frac{50 \times 10^6}{10^5}\right) = -30\ dB\mu V + 53.98\ dB = 23.98\ dB\mu V$$

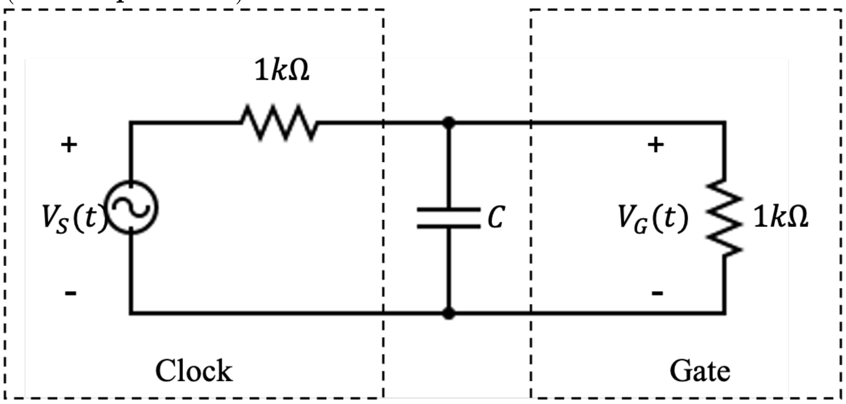
$$\mid Y(j\omega) \mid_{\omega=50\times10^6 rad/s} = 78.06\ dB\mu V + 23.98\ dB\mu V = \boxed{102.04\ dB\mu V}$$

4. A 5 V, 10 MHz oscillator having a rise/fall time of 10ns and a 50% duty cycle is applied to a gate shown below.

   a) Determine the value of the capacitance such that the fifth harmonic is reduced by 20 dB in the gate voltage $V_G(t)$.

   b) Using spectrum bound, estimate the level of this fifth harmonic in $dB\mu V$ at the source.

   c) (**Bonus problem**) Calculate the exact value of the fifth harmonic in $dB\mu V$ at the source.



Solution:

a)

Adding the capacitor will reduce the fifth harmonic is reduced by 20 dB:

$$20log_{10}\left(\frac{V_{G,with\ C}(f_5)}{V_{G,no\ C}(f_5)}\right) = -20\ dB$$

$$|\frac{V_{G,with\ C}(f_5)}{V_{G,no\ C}(f_5)}| = 10^{-20/20} = 0.1$$

From voltage division rule

$$V_{G,no\ C} = V_s \cdot \frac{1000}{1000+1000} = \frac{V_s}{2}$$

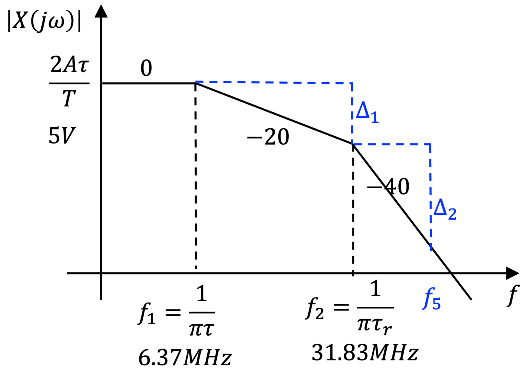
$$V_{G,with\ C} = V_s \cdot \frac{R_G//\frac{1}{j\omega C}}{R_S + R_G//\frac{1}{j\omega C}} = V_s \cdot \frac{\frac{R_G\frac{1}{j\omega C}}{R_G+\frac{1}{j\omega C}}}{R_S + \frac{R_G\frac{1}{j\omega C}}{R_G+\frac{1}{j\omega C}}} = V_s \cdot \frac{\frac{1000}{1000j\omega C+1}}{1000 + \frac{1000}{1000j\omega C+1}} = V_s \cdot \frac{1}{2+1000j\omega C}$$

Then the problem becomes

$$|\frac{2}{2+1000j\omega C}| = 0.1$$

Solve for capacitor value C with $f = 50MHz$ and $\omega = 2\pi f$, we get $C = \boxed{63.34pF}$

b)



period $T = \frac{1}{f} = \frac{1}{10MHz} = 0.1\mu s$
pulse width $\tau = DT = 50\% \times 0.1\mu s = 0.05\mu s$
$f_1 = \frac{1}{\pi\tau} = \frac{1}{\pi 0.05} MHz = 6.47MHz$
$f_2 = \frac{1}{\pi\tau_r} = \frac{1}{\pi\times 10\times 10^{-9}} = 31.83MHz$

$$\triangle_1 = -20log_{10}\left(\frac{f_2}{f_1}\right) = -20log_{10}\left(\frac{31.83}{6.37}\right) = -13.97\ dB$$

$$\triangle_2 = -40log_{10}\left(\frac{f_5}{f_2}\right) = -40log_{10}\left(\frac{50}{31.83}\right) = -7.85\ dB$$

$$Level_{f_5} = 20log_{10}\left(\frac{5}{1\mu V}\right) - \triangle_1 - \triangle_2 = 133.98 - 13.97 - 7.85 = \boxed{112.16\ dB\mu V}$$

c)

$$|C_5| = 2AD \cdot \frac{sin(n\omega_0\tau/2)}{n\omega_0\tau/2} \cdot \frac{sin(n\omega_0\tau_r/2)}{n\omega_0\tau_r/2}$$

$= 5 \cdot \frac{sin(5\times 2\pi\times 10^7\times 0.5\times 10^{-7}/2)}{5\times 2\pi\times 10^7\times 0.5\times 10^{-7}/2} \cdot \frac{sin(5\times 2\pi\times 10^7\times 10\times 10^{-9}/2)}{5\times 2\pi\times 10^7\times 10\times 10^{-9}/2}$
$= 5 \cdot \frac{sin(2.5\pi)}{2.5\pi} \cdot \frac{sin(0.5\pi)}{0.5\pi}$
$= 5 \cdot \frac{1}{7.854} \cdot \frac{1}{1.5708}$
$= 0.40528V$
$= \boxed{112.115\ dB\mu V}$

5. A typical coaxial cable is RG6U, which has an interior 18-gauge (radius 20.15 mils) solid wire, an interior shield radius of 90 mils (1 mil = 1/1000 inch), and an inner insulation of foamed polyethylene having a relative permittivity of $\epsilon_r = 1.45$. Determine the per-unit-length capacitance, inductance, and the velocity of propagation relative to that of free space.

   Solution:
   Given $r_1 = 20.15\ mil$, $r_2 = 90\ mil$, $\epsilon_1 = 1.45$, $\mu_0 = 4\pi \times 10^{-7}\ H/m$, $\epsilon_0 = 8.854 \times 10^{-12}\ F/m$, we have
   the per-unit-length inductance is

$$l = \frac{\mu_0 \mu_r}{2\pi} ln\left(\frac{r_2}{r_1}\right) = \frac{4\pi \times 10^{-7}}{2\pi} ln\left(\frac{90}{20.15}\right) = \boxed{299\ nF/m}$$

   the per-unit-length capacitance is

$$c = \frac{2\pi\epsilon_0\epsilon_r}{ln\left(\frac{r_2}{r_1}\right)} = \frac{2\pi \times 8.854 \times 10^{-12} \times 1.45}{ln\left(\frac{90}{20.15}\right)} = \boxed{53.9\ pH/m}$$

   the velocity is

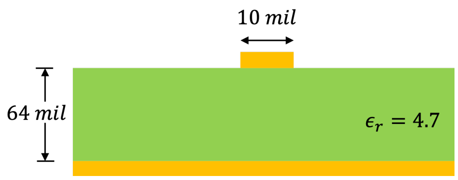
$$v = \frac{c_0}{\sqrt{\epsilon_r}} = \frac{c_0}{\sqrt{1.45}} = 0.83 c_0 = \boxed{2.49 \times 10^8\ m/s}$$

6. A microstrip line is constructed on a FR-4 board having a relative permittivity of 4.7. The board thickness is 64 mils and the trace width is 10 mils.

a) Determine the per-unit-length capacitance and inductance.

b) Determine the effective relative permittivity ϵ_e, characteristic impedance Z_0, and per-unit-length delay T_d.

Solution:



$$w = 10\ mil,\ \ h = 64\ mil$$
$$w/h \le 1$$

b)
The effective relative permitivity is approximately

$$\epsilon_r' = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\frac{1}{\sqrt{1 + 10h/w}} = 2.85 + \frac{3.7}{2}\frac{1}{\sqrt{1 + 10 \times 6.4}} = \boxed{3.079}$$

The characteristic impedance is

$$Z_c = \frac{60}{\sqrt{\epsilon_r'}} ln\left(\frac{8h}{w} + \frac{w}{4h}\right) = \frac{60}{\sqrt{3.079}} ln\left(\frac{8 \times 64}{10} + \frac{10}{4 \times 64}\right) = \boxed{134.59\ \Omega}$$

The propagation speed for the microstrip line is (c_0 is the speed of light)

$$v_p = \frac{c_0}{\sqrt{\epsilon_r'}} = \boxed{0.57c_0}$$

The per-unit-length delay is

$$T_d = 1/v_p = \boxed{5.85\ ns/m}$$

a)
So the per-unit-length capacitance is

$$c = \frac{1}{v_p Z_0} = \frac{1}{0.57 \times 3 \times 10^8 \times 134.59} = \boxed{43.45\ pF/m}$$

The per-unit-length inductance is

$$l = \frac{\mu_0 \epsilon_0 \epsilon_r'}{c} = \frac{\epsilon_r'}{c_0^2 c} = \frac{3.079}{(3 \times 10^8)^2 \times 43.45 \times 10^{-12}} = \boxed{787.36\ nH/m}$$