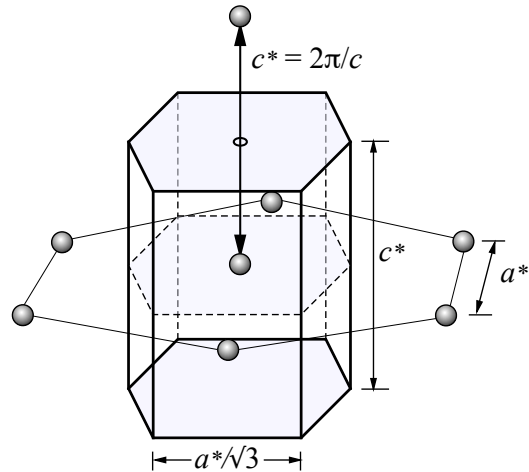


**Fig. 2-45** Construction of the first Brillouin zone for the plane simple hexagonal lattice. The reciprocal lattice points are shown at six corners and the origin.



**Fig. 2-46** First Brillouin zone for the simple hexagonal lattice. Some lattice points of the reciprocal lattice are also shown.

**(c) 3D Brillouin zone of a simple hexagonal crystal:**

The reciprocal lattice of a simple hexagonal lattice is another simple hexagonal lattice rotated  $30^\circ$  about the  $c$ -axis. If the basic lattice translations for the direct lattice in the basal plane and along the  $c$ -axis are respectively  $a$  and  $c$  then for the reciprocal lattice they are  $a^*$  and  $c^*$ , where  $a^* = (4\pi/\sqrt{3})/a$  and  $c^* = 2\pi/c$ . Figure 2-45 shows seven points of the reciprocal lattice of a simple hexagonal lattice. If we take the central point  $O$  as origin, the other six lie at the corners of a regular hexagon of side  $a^* = (4\pi/\sqrt{3})/a$ , and are the six nearest neighbors. The first Brillouin zone is formed by connecting bisectors of the lines joining  $O$  to the other six nearest neighbors. It is a hexagon whose sides have length  $a^*/\sqrt{3} = 4\pi/3a$  and hence covers a unit cell area of  $(a^*)^2\sqrt{3}/2 = (8\pi^2/\sqrt{3})/a^2$ . It should be noted that the hexagon forming the first Brillouin zone has the same orientation as the hexagons of the direct lattice.

The 3D first Brillouin zone of the simple hexagonal lattice is obtained simply by introducing planes parallel to the  $c$ -axis through the sides of the hexagon and two planes perpendicular to the  $c$ -axis each at distance  $c^*/2$  from  $O$ . The first Brillouin zone is therefore a hexagonal prism of height  $c^* = 2\pi/c$  whose axis is parallel to the  $c$ -axis of the direct lattice. The hexagonal of side  $4\pi/3a$  is oriented in the same direction as the hexagons of the direct lattice. It occupies a volume of  $(16\pi^3/\sqrt{3})/a^2c$ . The first Brillouin zone is shown in Fig. 2-46, together with some points of the reciprocal lattice.