**ECE 486** **Assignment # 1**

Issued: Jan 18 Due: Jan 25, 2024

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### Reading Assignment:

**FPE**, 6th ed., Sections 1.1, 1.2, 2.1–2.4, 7.2, 9.2.1.

### Problems:

(the first two problems are designed to test your background)

1. Compute the characteristic polynomial $P(\lambda) = \det(A - \lambda I)$ and the eigenvalues of each matrix $A$ given below:

   (i) $A = \begin{pmatrix} 1 & 4 \\ 4 & 2 \end{pmatrix}$

   (ii) $A = \begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix}$

   (iii) $A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$

2. Compute the magnitude and the phase of the following complex numbers:

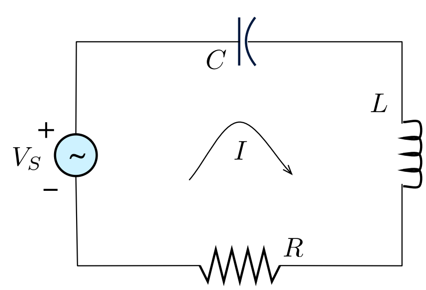
   (i) $3 + 2j$

   (ii) $2 - j$

   (iii) $\dfrac{3 + 2j}{2 - j}$

   How do the answers for (iii) relate to those for (i) and (ii)? State the general rule behind this.

3. Derive a state-variable model, of the form $\dot{x} = Ax + Bu$, of the following circuit:




   Note that you have to decide which variables to take as the states and which one to take as the input. Make sure to declare your choice.

4. Convert each of the following high-order differential equations into the state-variable form:

   (i) $\ddot{x} + \dot{x} = -u$

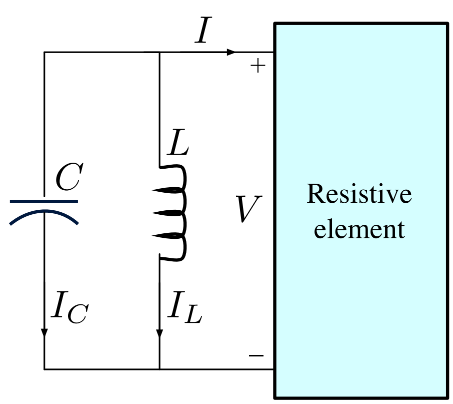
   (ii) $x^{(3)} + \ddot{x} - x = u$ ($x^{(3)}$ is the 3rd derivative of $x$ with respect to time)

5. An *autonomous* nonlinear state-space model is a system of first-order ODEs that has the form

$$\dot{x} = f(x),$$

where $x \in \mathbb{R}^n$ is the state vector. The term “autonomous” designates the fact that the external input $u$ is absent, so the system evolves autonomously, or on its own. We say that a point $x_0 \in \mathbb{R}^n$ is an *equilibrium point* of the system if $f(x_0) = 0$.

Consider the following circuit that contains linear components (an inductor and a capacitor) and a nonlinear resistive element:




The voltage $V$ across the resistive element and the current $I$ flowing into it are related via a nonlinear voltage-current characteristic $I = g(V)$.

   (i) Derive a second-order ODE for $V$. You may (and should) assume that $g$ is differentiable.

   (ii) Write down an autonomous nonlinear state-space model for the ODE you have obtained in part (i).

   (iii) Consider the following voltage-current characteristic:

$$g(V) = -V + \tfrac{1}{3}V^3.$$

   Show that the zero state is the only eqiulibrium point of the state-space model from part (ii) and linearize it.