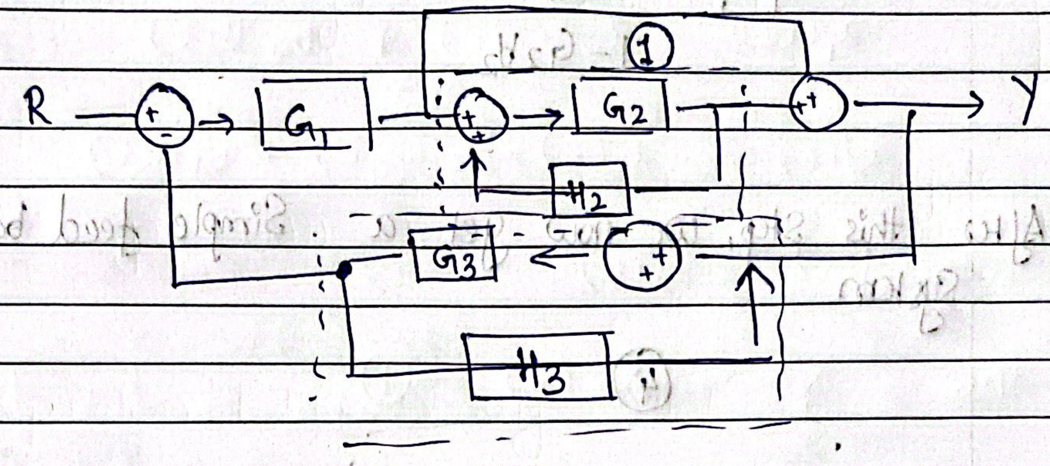
ECE 486 – HW2 Solution

Q1

(a)



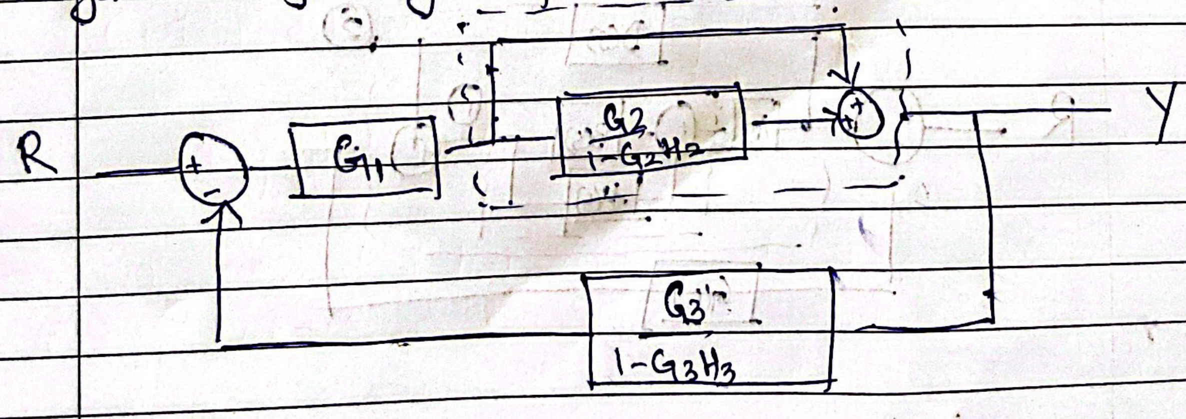
1) We can simplify (1) using Block diagram Rules

$$\frac{G_2}{1-G_2H_2}$$

2) We can further Simply (2) as follows,

$$\frac{G_3}{1-G_3H_3}$$

We get the following Simplification (3)

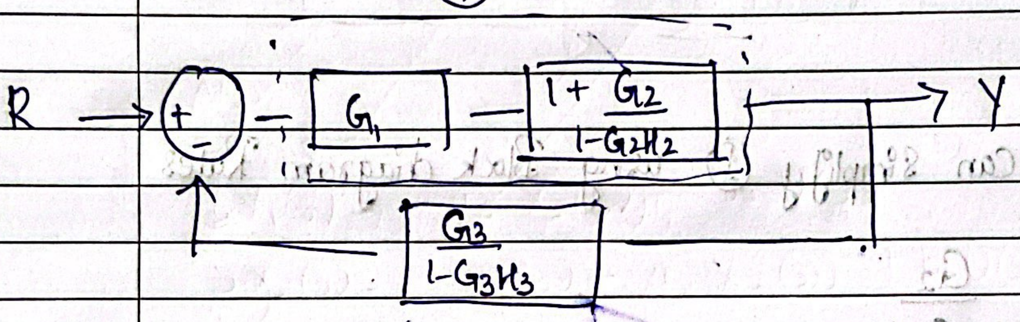


we can now simplify (3) as

$$1 + \frac{G_2}{1 - G_2 H_2}$$

After this step, we now get a simple feed back system

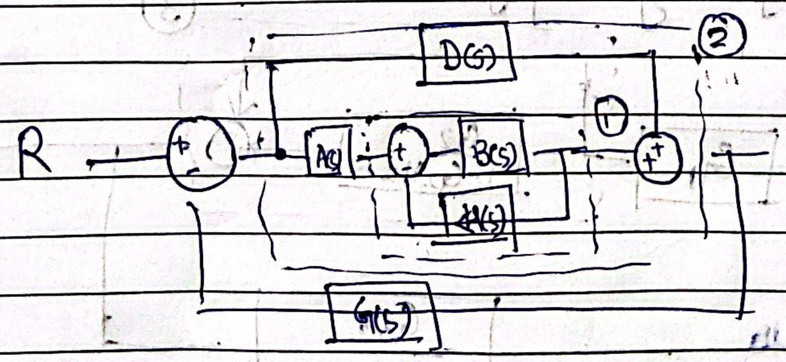
(4)



(4) is a simple feedback system such that

$$\frac{Y(s)}{R(s)} = \frac{G_1\left(1 + \frac{G_2}{1-G_2H_2}\right)}{1 + G_1\left(1 + \frac{G_2}{1-G_2H_2}\right)\left(\frac{G_3}{1-G_3H_3}\right)}$$

(6)

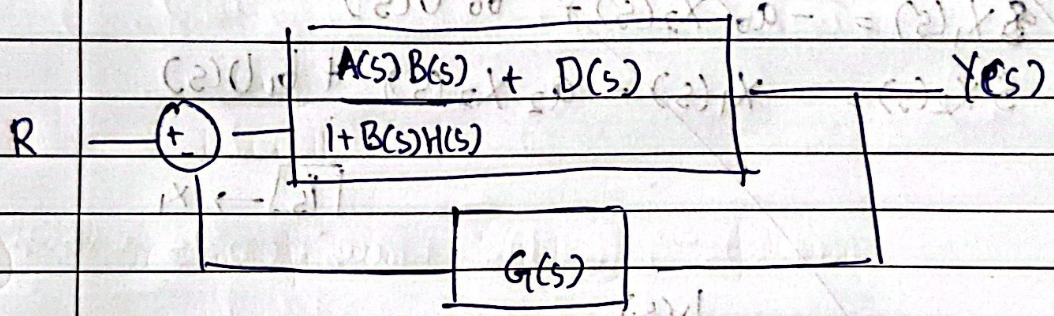


Simplifying ① and ② gives us,

① $\dfrac{B(s)}{1+B(s)H(s)}$

② $\dfrac{A(s)\cdot B(s)}{1+B(s)H(s)} + D(s)$

We now get,



Simplifying this using Feedback Loop formula gives us

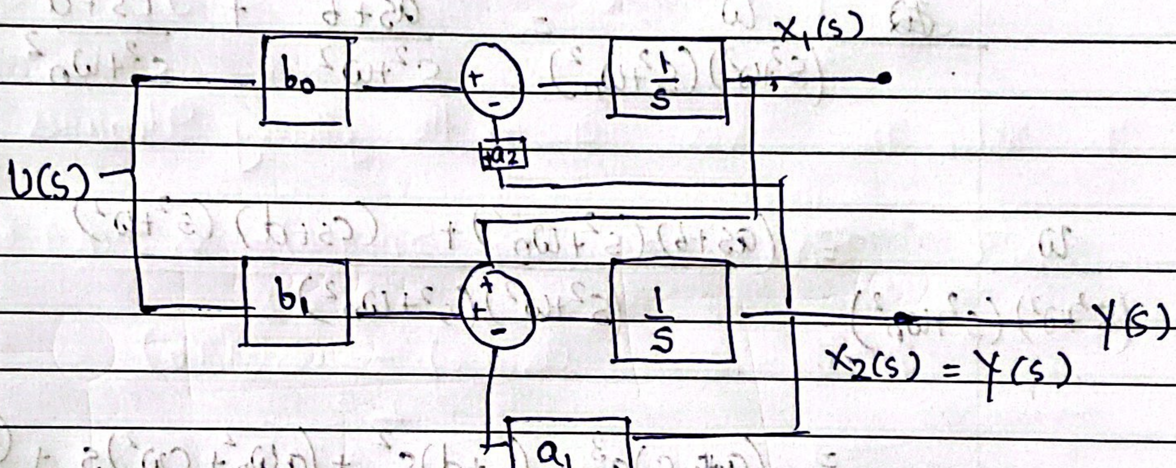
$$\frac{Y(s)}{R(s)} = \frac{\left(\dfrac{A(s)B(s)}{1+B(s)H(s)} + D(s)\right)}{1 + G(s)\left(\dfrac{A(s)B(s)}{1+B(s)H(s)} + D(s)\right)}$$

Q2

$$\dot{x}_1(t) = -a_2 x_2(t) + b_0 u(t)$$

$$\dot{x}_2(t) = x_1(t) - a_1 x_2(t) + b_1 u(t)$$

$$y(t) = x_2(t)$$



Note that this is one of the Solutions. Different Solution might exist

Q3

$$H(s) = \frac{1}{s^2+\omega_n^2} \qquad U(s) = \frac{\omega}{s^2+\omega^2}$$

Thus, $Y(s) = H(s) \cdot U(s) = \dfrac{\omega}{(s^2+\omega_n^2)(s^2+\omega^2)}$

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This can be ~~made~~ Solved for Laplace Inverse as follows

$$\mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}\left(\frac{\omega}{(s^2+\omega^2)(s^2+\omega_n^2)}\right)$$

Using partial fractions

$$\frac{\omega}{(s^2+\omega^2)(s^2+\omega_n^2)} = \frac{as+b}{s^2+\omega^2} + \frac{cs+d}{s^2+\omega_n^2}$$

$$\frac{\omega}{(s^2+\omega^2)(s^2+\omega_n^2)} = \frac{(as+b)(s^2+\omega_n^2)+(cs+d)(s^2+\omega^2)}{(s^2+\omega^2)(s^2+\omega_n^2)}$$

$$= \frac{(a+c)s^3+(b+d)s^2+(a\omega_n^2+c\omega^2)s+(b\omega_n^2+d\omega^2)}{(s^2+\omega^2)(s^2+\omega_n^2)}$$

Comparing Numerator on both sides, we get

$a+c=0 \qquad b+d=0 \qquad a\omega_n^2+c\omega^2=0$

$b\omega_n^2+d\omega^2=\omega$

$a=c=0 \qquad b(\omega_n^2-\omega^2)=\omega$

$$b=\frac{\omega}{\omega_n^2-\omega^2} \qquad d=\frac{-\omega}{\omega_n^2-\omega^2}$$

$$\mathcal{L}^{-1}(Y(s)) = \mathcal{L}\left(\frac{b}{s^2+\omega^2} + \frac{d}{s^2+\omega_n^2}\right)$$

$$= \frac{b}{\omega}\sin(\omega t) + \frac{d}{\omega}\sin(\omega_n t)$$

$$Y(t) = \frac{1}{\omega_n^2-\omega^2}\left(\sin(\omega t) - \sin(\omega_n t)\right)$$

$\lim_{\omega\to\omega_n} y(t) \to \infty$ (since there is denominator term that blows up)

~~We can~~ We can see the same thing by observing magnitude for a Bode plot.

$$|G(j\omega)| = \left|\frac{1}{\omega^2-\omega_n^2}\right|$$

$$\lim_{\omega\to\omega_n} |G(j\omega)| \to \infty$$

(b) Using Convolution, we see that

$$y(t) = \int_0^t \frac{\sin(\omega_n(t-\tau))}{\omega_n} \cdot \sin\omega t \, dt$$

$$y(t) = \frac{1}{\omega_n}\int_0^t \sin(\omega_n(t-\tau)) \cdot \sin(\omega\tau)\, d\tau$$

$$= \frac{1}{\omega_n}\int_0^t \left(\sin(\omega_n t)\cos(\omega_n \tau) - \cos(\omega_n t)\sin(\omega_n \tau)\right) \sin(\omega\tau) \cdot d\tau$$

$$= \frac{1}{\omega_n}\int_0^T \sin(\omega_n t)\left(\sin(\omega\tau)\cos\omega_n\tau\right)d\tau - \int (\cos\omega_n t)(\sin\omega_n\tau \sin(\omega\tau)\, d\tau)$$

To Solve this, ~~Cos~~ Consider the following Identities

$$\sin a \cos b = \frac{\sin(a+b) + \sin(a-b)}{2}$$

$$\sin a \sin b = \frac{\cos(a-b) - \cos(a+b)}{2}$$

$$\frac{1}{\omega_n}\left\{ \sin(\omega_n t)\int_0^t \frac{\sin((\omega+\omega_n)\tau)d\tau + \sin((\omega-\omega_n)\tau)\, d\tau}{2} \right.$$

$$\left. - \cos(\omega_n t)\int_0^t \frac{\cos((\omega-\omega_n)\tau) - \cos((\omega+\omega_n)\tau)\, d\tau}{2} \right\}$$

$$\frac{1}{\omega_n}\left(\sin(\omega_n t)\left(-\frac{\cos(\omega+\omega_n)}{(\omega+\omega_n)}\Bigg|_0^t - \frac{\cos(\omega-\omega_n)t}{(\omega-\omega_n)}\Bigg|_0^t \right)\right)$$

$$-\frac{1}{\omega_n}\left(\cos(\omega_n t)\left(\frac{\sin(\omega-\omega_n t)}{\omega-\omega_n}\Bigg|_0^t - \frac{\sin((\omega+\omega_n)t)}{\omega+\omega_n}\Bigg|_0^t \right)\right)$$

as $\omega \to \omega_n$ the above term blows up

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Q4

(a) We use Routh - Horwitz Criteria

$s^4 + 8s^3 + 32s^2 + 80s + 10$

Cond 1: All Coefficients are positive

Cond 2:

s^4	1	32	10
s^3	8	80	0
s^2	$\frac{170}{8}$	10	
s^1	$\frac{1620 \times 8}{17}$	0	
s^0	$\frac{1620 \times 8}{17}$, 10		
	$\frac{162 \times 8}{17}$		

→ Since, sign changes = 0

→ No Root on RHS plane

(b) $s^5 + 5s^4 + 2s^3 - s^2 + 4s + 10$

Since, one coefficient is negative, there are Roots in RHS