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- Review: control design using frequency response: PI/lead
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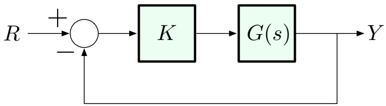
Reading: FPE, Chapter 6

Review: Bode's Gain-Phase Relationship



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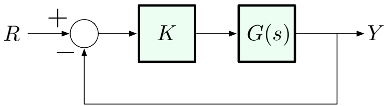

Assuming that $G(s)$ is *minimum-phase* (i.e., has no RHP zeros), we derived the following for the Bode plot of $KG(s)$:

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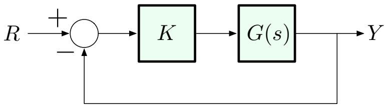

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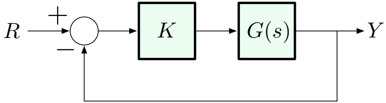

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We can state this succinctly as follows:

Gain-Phase Relationship. Far enough from break-points,

$$\text{Phase} \approx \text{Magnitude Slope} \times 90°$$

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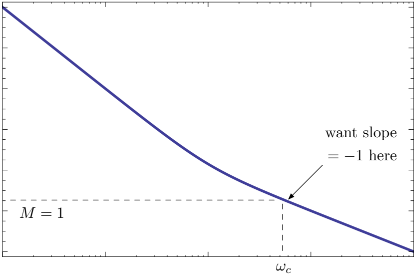

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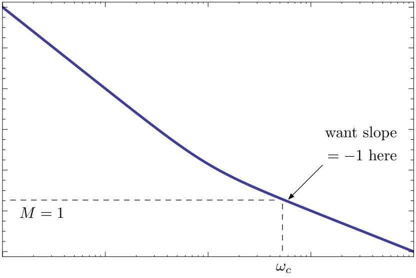

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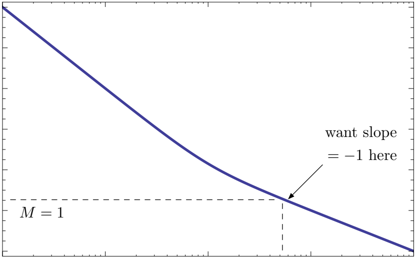

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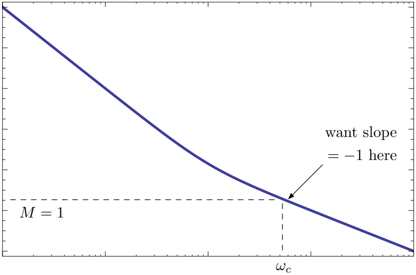

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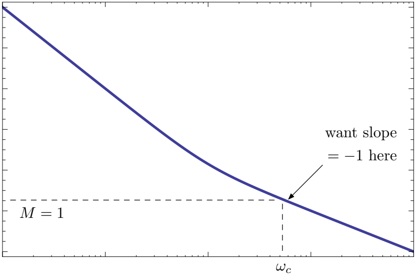

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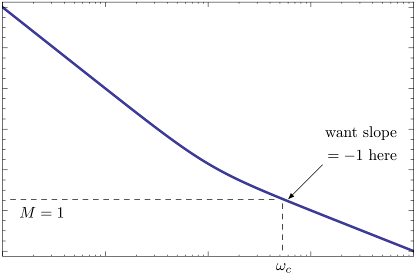

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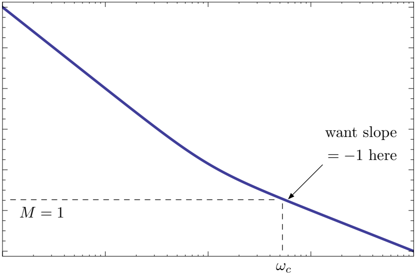

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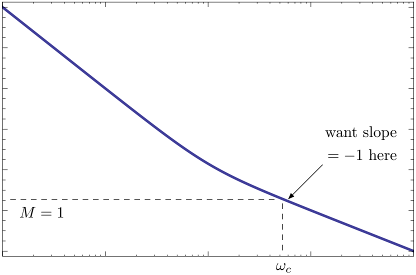

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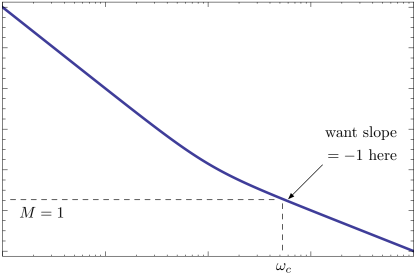

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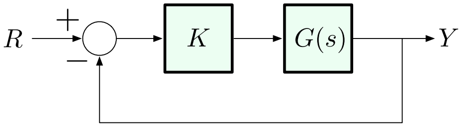
(Similar considerations apply when M-plot has positive slope – depends on the t.f.)

Control Design Using Frequency Response



Bode's Gain-Phase Relationship suggests that we can shape the time response of the *closed-loop* system by choosing K (or, more generally, a dynamic controller $KD(s)$) to tune the Phase Margin.

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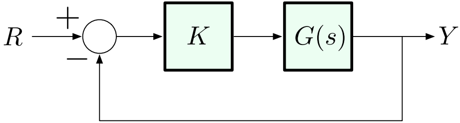

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In particular, from the quantitative Gain-Phase Relationship,

$$\text{Magnitude slope}(\omega_c) = -1 \qquad \Longrightarrow \qquad \text{Phase}(\omega_c) \approx -90^\circ$$

— which gives us PM of 90° and consequently good damping.

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This is an intuitive procedure, but it's not very precise, requires trial & error.

Lag Compensation: Bode Plot

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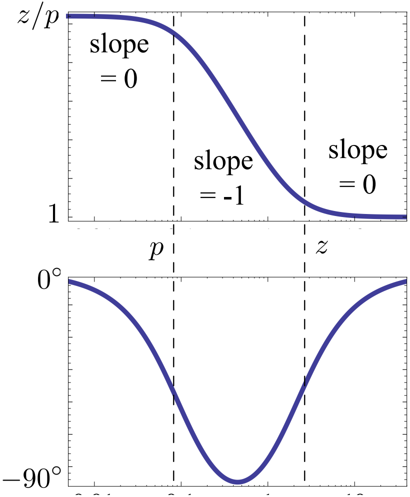

- $\frac{j\omega+z}{j\omega+p} \xrightarrow{\omega\to\infty} 1$
  so $M \to 1$ at high frequencies

- subtracts phase, hence the term "phase lag"

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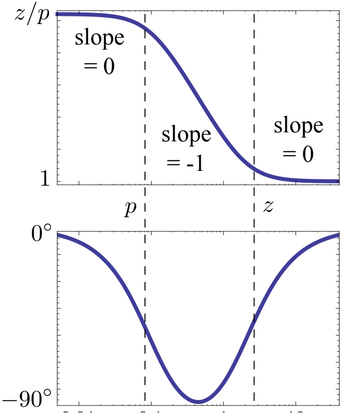

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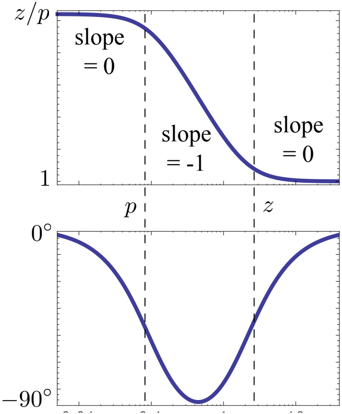

- $$\frac{j\omega + z}{j\omega + p} \xrightarrow{\omega \to 0} \frac{z}{p}$$

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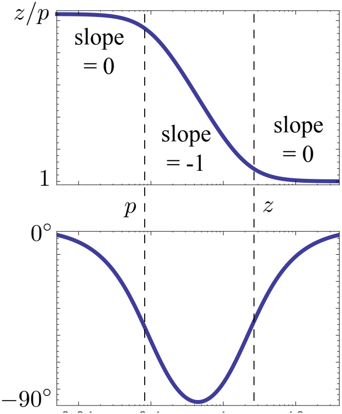

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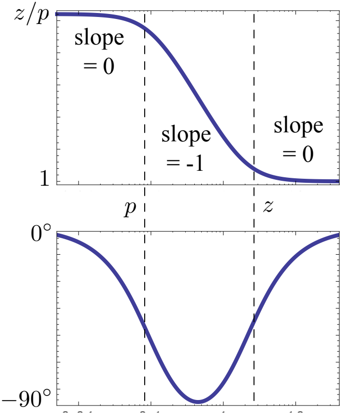

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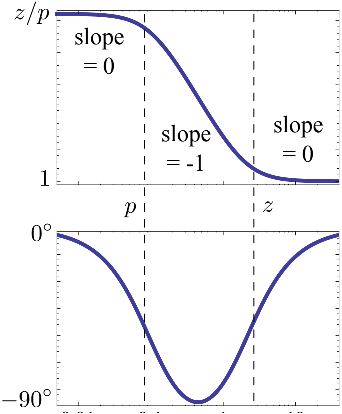

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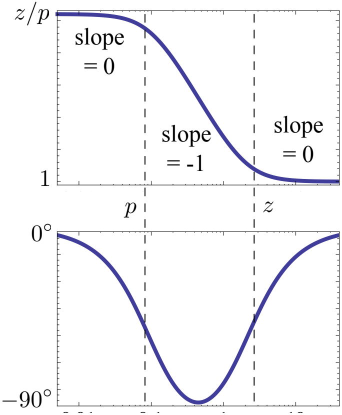

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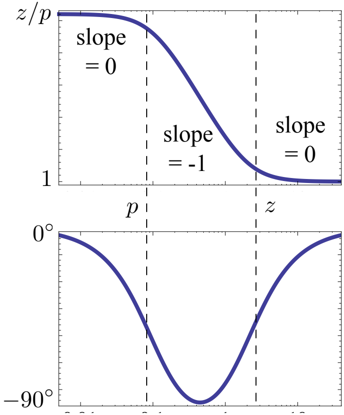

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- caution: lead increases PM, but adding lag can undo this
- to mitigate this, choose both z and p very small, while maintaining desired ratio z/p

Example

$$G(s) = \frac{1}{(s+0.2)(s+0.5)} \overset{\text{Bode form}}{=} \frac{10}{\left(\frac{s}{0.2}+1\right)\left(\frac{s}{0.5}+1\right)}$$

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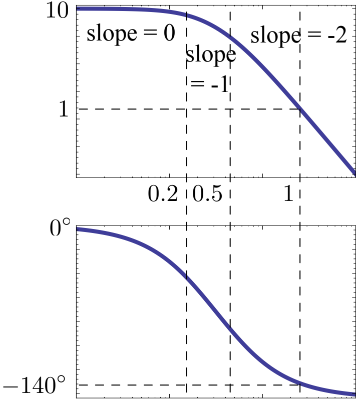
- z and p will be chosen to get good tracking
- PM will be shaped by choosing K
- this is different from what we did for lead (used p and z to shape PM, then chose K to get desired bandwidth spec)

Step 1: Choose K to Shape PM

Check Bode plot of $G(s)$ to see how much PM it already has:



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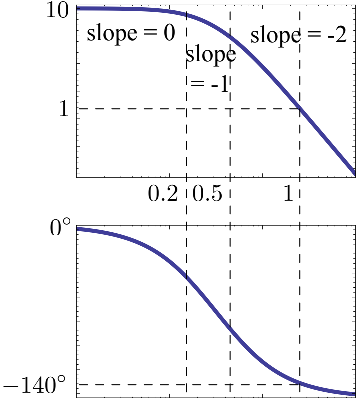

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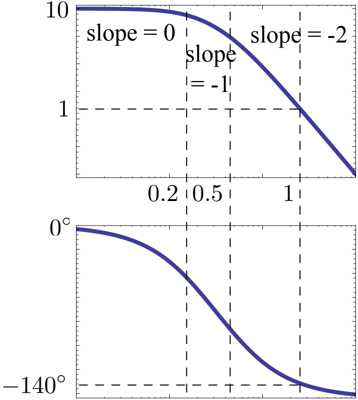

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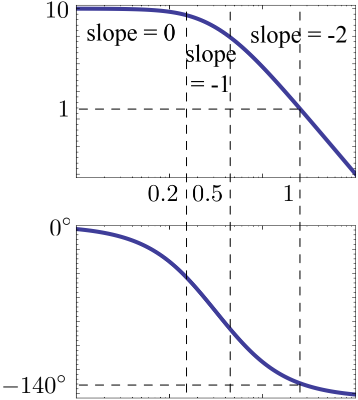

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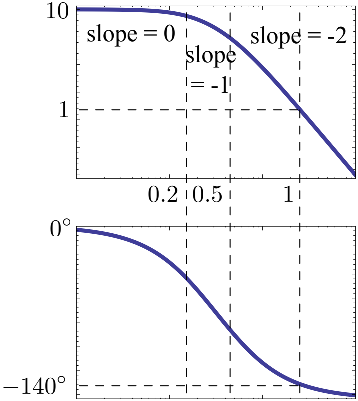

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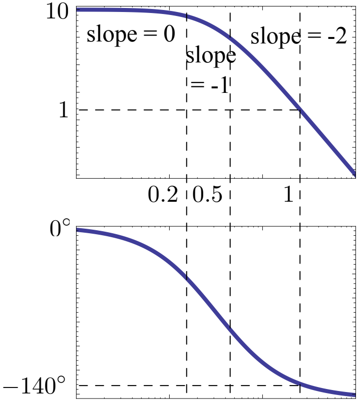

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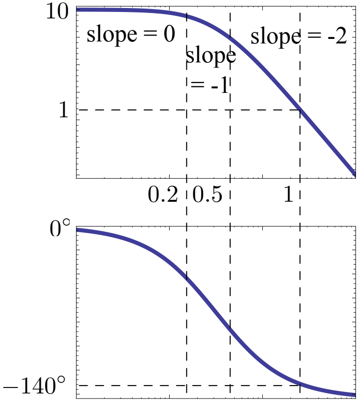

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A conservative choice (to allow some slack) is $K = 1/2.5 = 0.4$, gives $\omega_c \approx 0.52$, PM $\approx 65°$

Step 2: Choose z & p to Shape Tracking Error

So far: $KG(s) = \dfrac{0.4 \cdot 10}{\left(\frac{s}{0.2} + 1\right)\left(\frac{s}{0.5} + 1\right)}$

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$$e(\infty) = \left.\frac{1}{1 + KG(s)}\right|_{s=0} = \frac{1}{1+4} = \frac{1}{5} = 20\% \quad \text{(too high)}$$

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$$e(\infty) = \left.\frac{1}{1+KG(s)}\right|_{s=0} = \frac{1}{1+4} = \frac{1}{5} = 20\% \quad \text{(too high)}$$

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$$D(0) = \left.\frac{s+z}{s+p}\right|_{s=0} = \frac{z}{p} \ge \frac{9}{4} = 2.25 \qquad \text{— say, } z/p = 2.5$$

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Not to distort PM and ω_c, let's pick z and p an order of magnitude smaller than $\omega_c \approx 0.5$: $z = 0.05$, $p = 0.02$

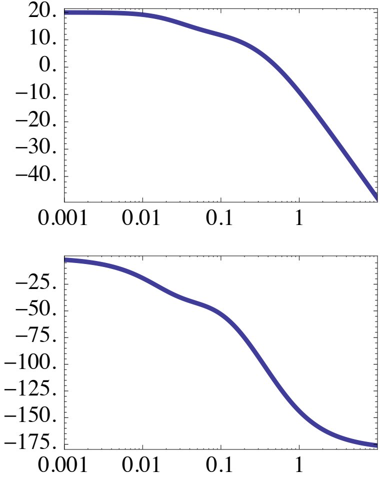
Overall Design

Plant:

$$G(s) = \frac{10}{\left(\frac{s}{0.2}+1\right)\left(\frac{s}{0.5}+1\right)}$$

Controller:

$$KD(s) = 0.4\frac{s+0.05}{s+0.02}$$

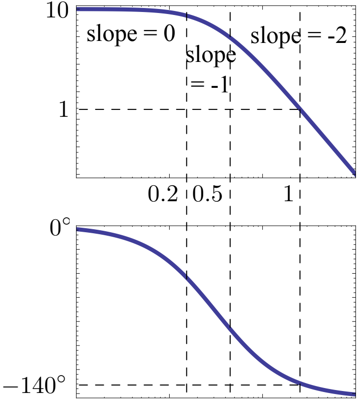


— the design still needs a bit of refinement ...

Lead & Lag Compensation

Let's combine the advantages of PD/lead and PI/lag.

Back to our example: $G(s) = \dfrac{10}{\left(\frac{s}{0.2}+1\right)\left(\frac{s}{0.5}+1\right)}$



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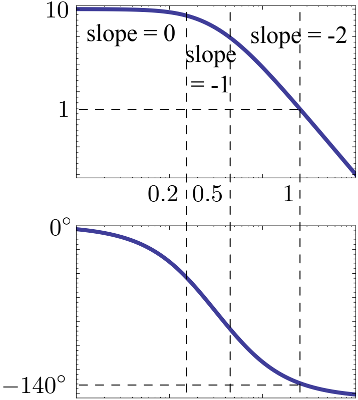

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New objectives:

- $\omega_{\mathrm{BW}} \geq 2$
- PM $\geq 60^\circ$
- $e(\infty) \leq 1\%$ for const. ref.

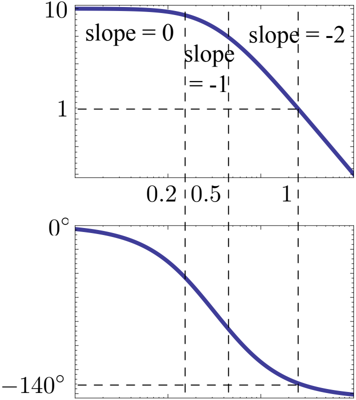
Lead & Lag Compensation

What we got before, with lag only:

- Improved PM by adjusting K to decrease ω_c.
- This gave $\omega_c \approx 0.5$, whereas now we want a larger ω_c (recall: $\omega_{\text{BW}} \in [\omega_c, 2\omega_c]$, so $\omega_c = 0.5$ is too small)

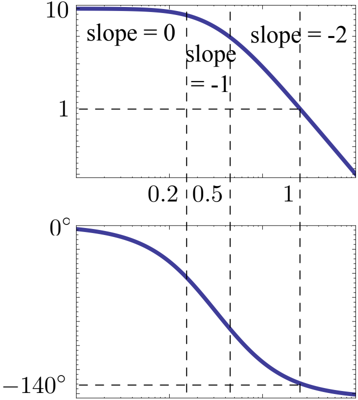
So: we need to reshape the phase curve using lead.

Lead & Lag Compensation



Step 1. Choose K to get $\omega_c \approx 2$ (before lead)

Lead & Lag Compensation



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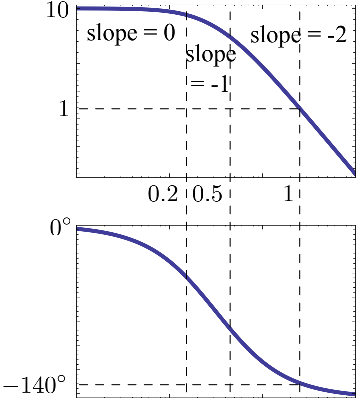

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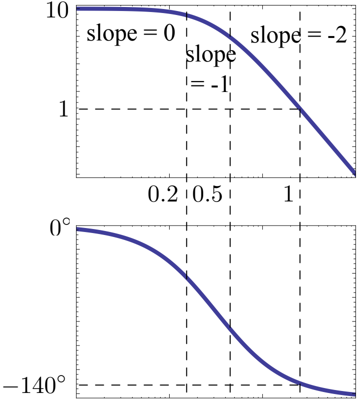

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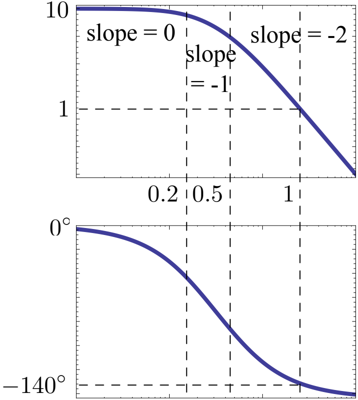

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— choose $K = 4$

(gives ω_c slightly < 2, but still ok).

Lead & Lag Compensation

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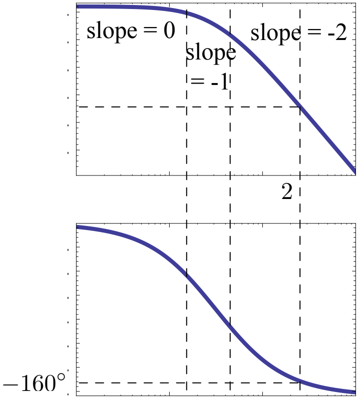

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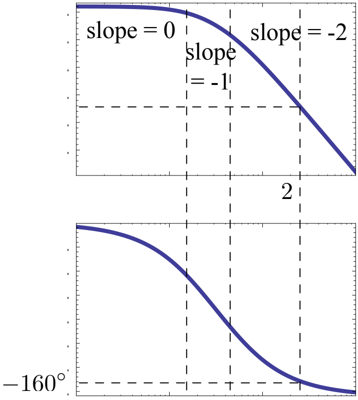

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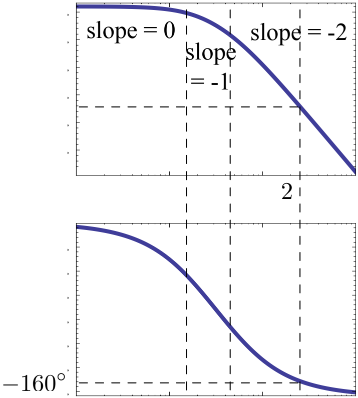

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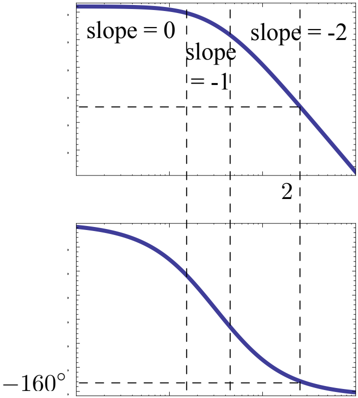

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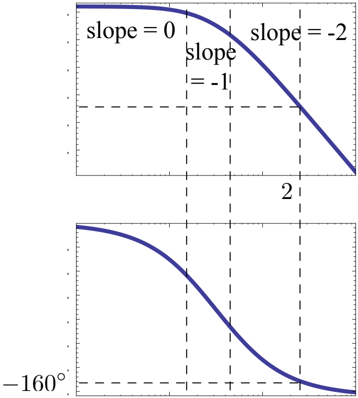

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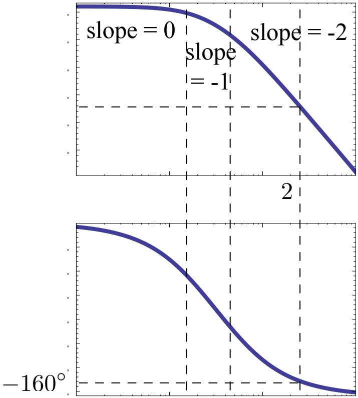

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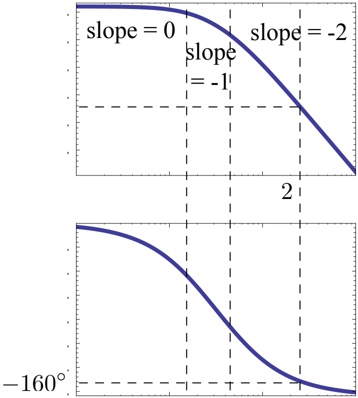

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The choice of lead pole/zero must satisfy

$$\sqrt{z_{\text{lead}} \cdot p_{\text{lead}}} \approx 2 \implies z_{\text{lead}} \cdot p_{\text{lead}} = 4$$

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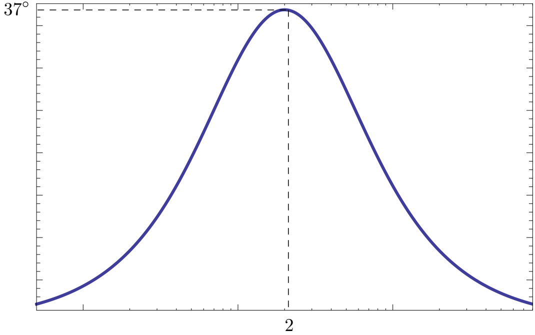

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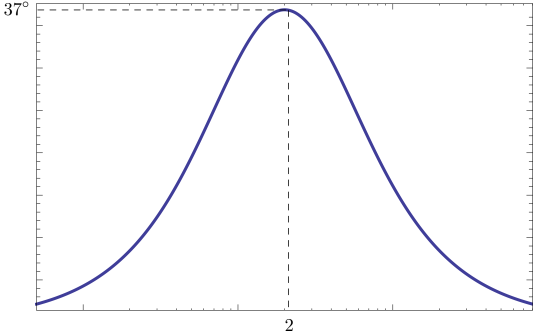

Phase lead = 37° — not enough!!

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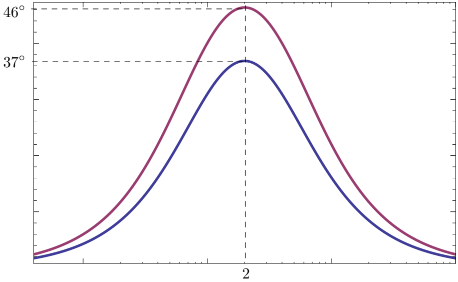
Need to space z_{lead} and p_{lead} farther apart:

$$\begin{cases} z_{\text{lead}} = 0.8 \\ p_{\text{lead}} = 5 \end{cases} \implies \quad \text{phase lead } = 46°$$



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So far:

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We want $D(0) \geq \frac{99}{40}$ with lag $\qquad \frac{z_{\text{lag}}}{p_{\text{lag}}} \approx 2.5$ will do

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Overall controller:

$$\underbrace{4\frac{\frac{s}{0.8}+1}{\frac{s}{5}+1}}_{\substack{\text{lead (with}\\ \text{gain } K=4 \text{ absorbed)}}} \cdot \underbrace{\frac{s+0.05}{s+0.02}}_{\substack{\text{lag (not in}\\ \text{Bode form)}}}$$

(Note: we don't rewrite lag in Bode form, because $z_{\text{lag}}/p_{\text{lag}}$ is not incorporated into K.)

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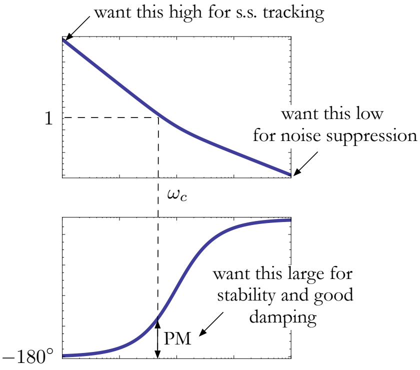

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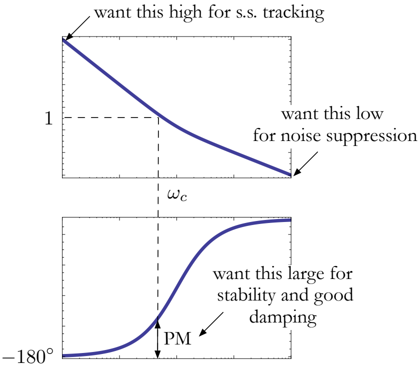

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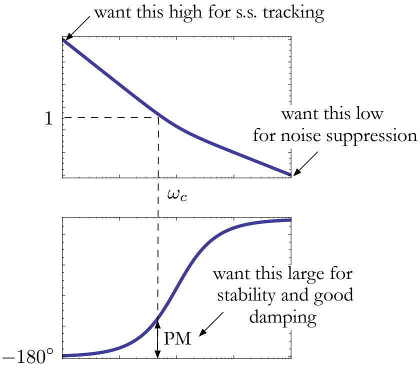

- evaluating the design and seeing which way to change it
- using experimental data (frequency response of the uncontrolled system can be measured experimentally)

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What we want is a frequency-domain substitute for the Routh–Hurwitz criterion — this is the Nyquist criterion, which we will discuss in the next lecture.