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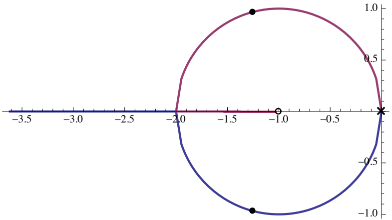
Reading: FPE, Chapter 5

From Last Time: Double Integrator with PD-Control

Characteristic equation: $\quad 1 + K \cdot \dfrac{s+1}{s^2} = 0$



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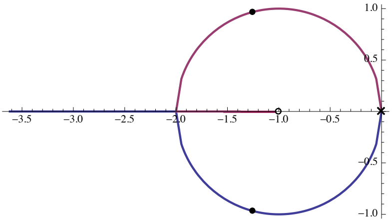

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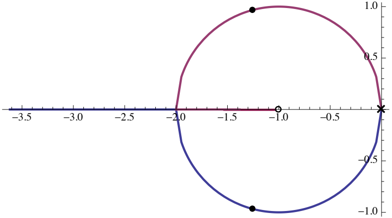

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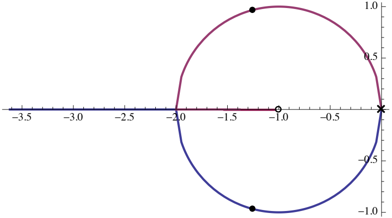

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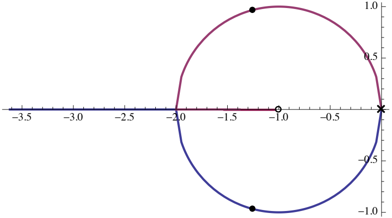

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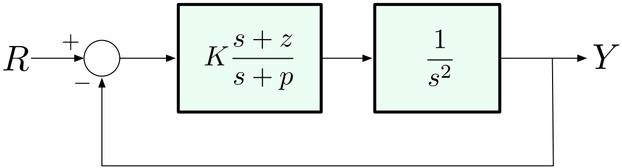
So, the effect of D-gain was to introduce an *open-loop zero* into LHP, and this zero "pulled" the root locus into LHP, thus stabilizing the system.

Dynamic Compensation

Objectives: stabilize the system and satisfy given time response specs using a *stable*, *causal* controller.



Characteristic equation:

$$1 + K \cdot \frac{s+z}{s+p} \cdot \frac{1}{s^2} = 1 + KL(s) = 0$$

Approximate PD Using Dynamic Compensation

Reminder: we can approximate the D-controller $K_{\rm D}s$ by

$$K_{\rm D}\frac{ps}{s+p} \longrightarrow K_{\rm D}s \text{ as } p \to \infty$$

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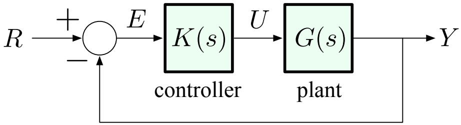

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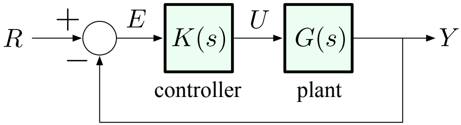

Closed-loop poles: $1 + \left(K_{\mathrm{P}} + K_{\mathrm{D}}\frac{ps}{s+p}\right)G(s) = 0$

Lead & Lag Compensators

Consider a general controller of the form

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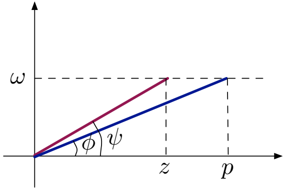
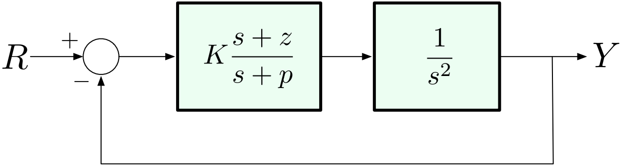
- if $z < p$, then $\psi - \phi > 0$ (phase lead)
- if $z > p$, then $\psi - \phi < 0$ (phase lag)



Back to Double Integrator



Controller transfer function is $K\dfrac{s+z}{s+p}$, where:

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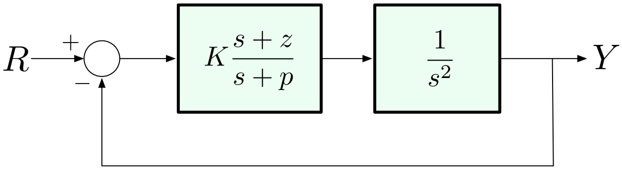

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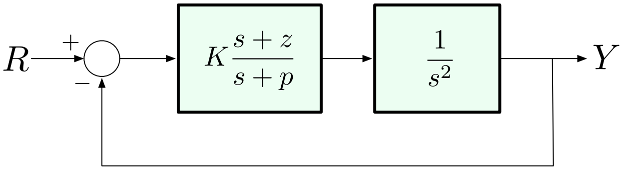

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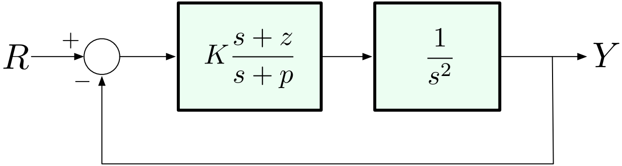

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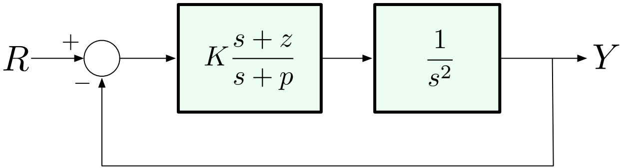
We use lead controllers as dynamic compensators for approximate PD control.

Double Integrator & Lead Compensator



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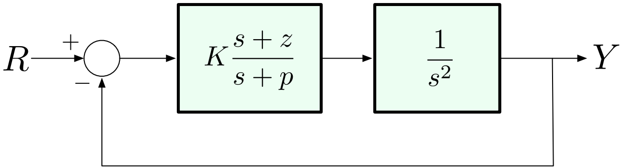

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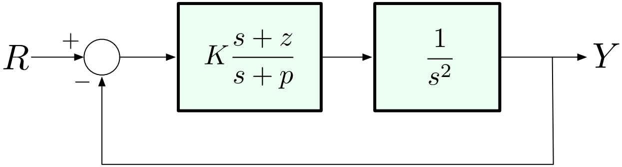

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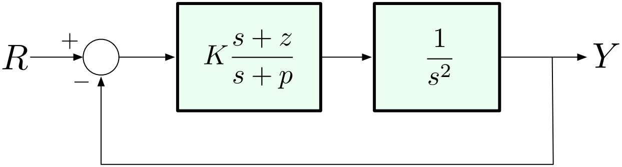

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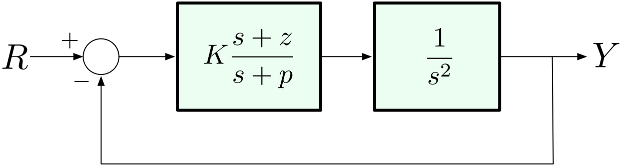

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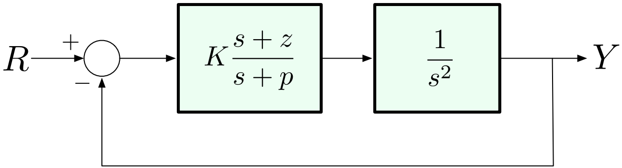

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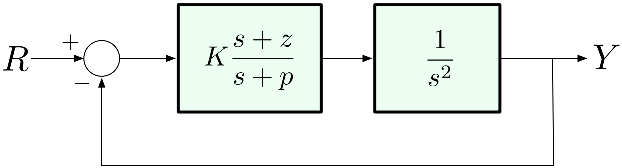

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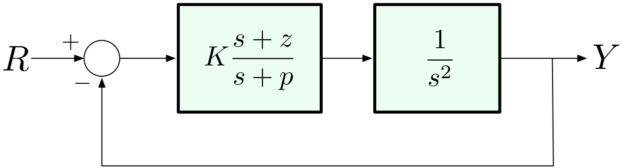

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We expect to get behavior similar to PD control.

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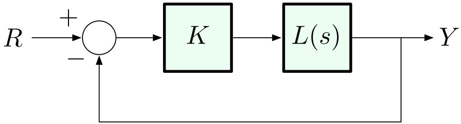

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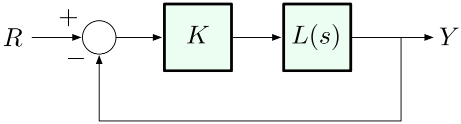

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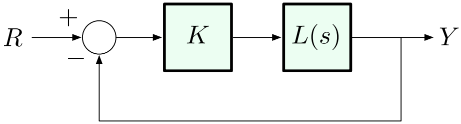

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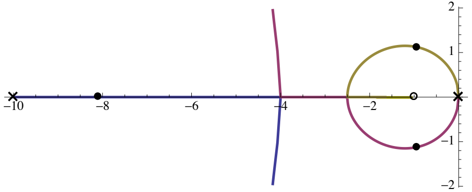

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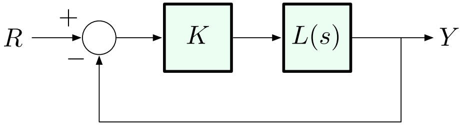

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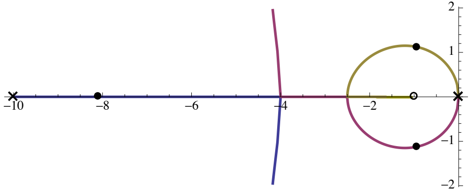

Close to $j\omega$-axis, this root locus looks similar to the PD root locus. However, the pole at $s = -10$ makes the locus look different for s far into LHP.

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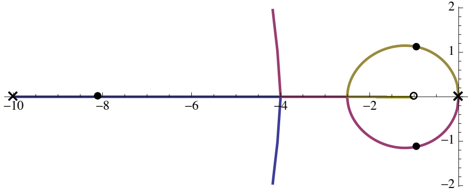
Root locus for $p = 10$:



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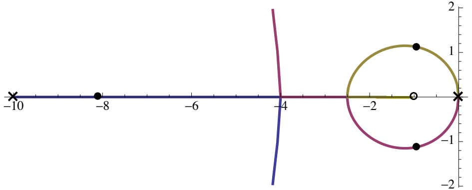

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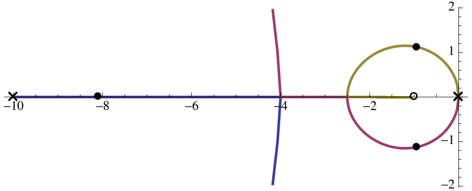

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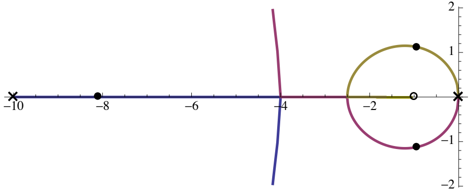

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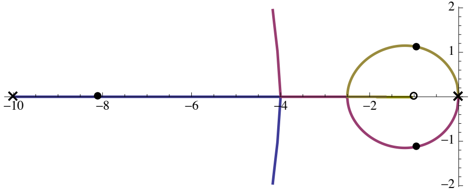

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(This is just intuition for now — we will confirm it later using frequency-domain methods.)

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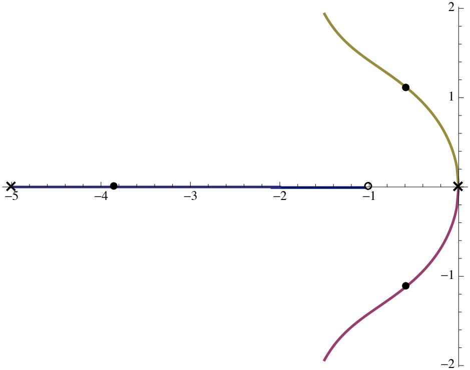

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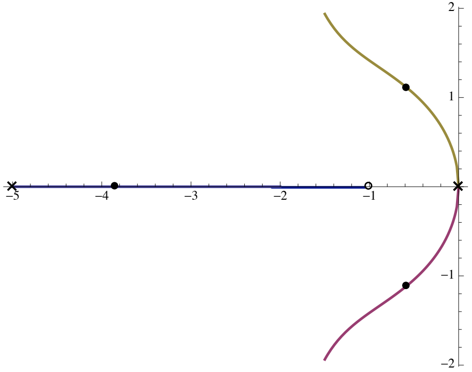

— for this value of p, the root locus is different, not nearly as nicely damped as for $p = 10$.

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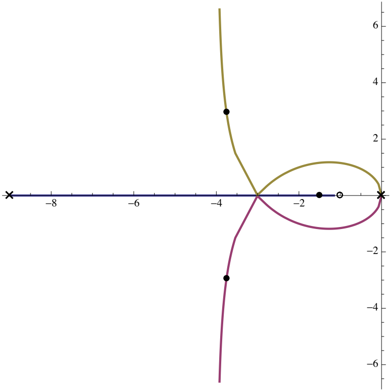

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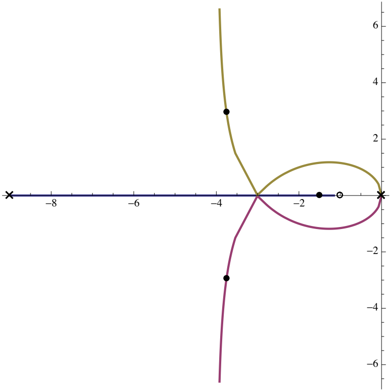
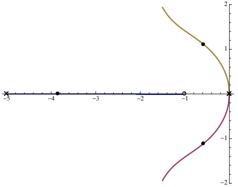


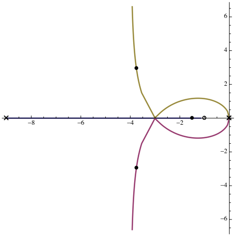
— for this value of p, the branches meet (*break in*) and separate (*break away*) at the same point on the real axis.

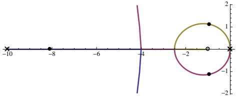
Summary on Design Trade-offs

From what we have seen so far:

- p large — good damping, but bad noise suppression (too close to PD); the branches first break in (meet at the real axis), then break away.
- p small — noise suppression is better, but RL is too close to $j\omega$-axis, which is not good; no break-in for small values of p.
- intermediate values of p — transition between two types of RL; break-in and break-away points are the same.







Lead Controller Design

With a lead controller in place, we have

$$KL(s) = K\frac{s+z}{s+p} \cdot G_p(s)$$

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Is there a systematic procedure for doing this?

Pole Placement Using RL

Back to our example: double integrator with lead compensation

$$KL(s) = K\frac{s+z}{s+p} \cdot \frac{1}{s^2}$$

Problem: given p and a desired closed-loop pole s, find the value of z that will guarantee this (if possible).

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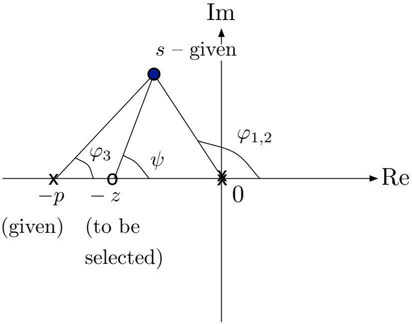

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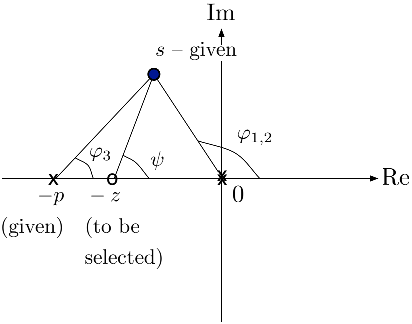
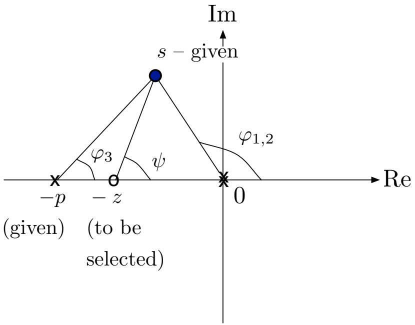


Must have

$$\underbrace{\psi}_{\text{angle from } s \text{ to zero}} - \sum_i \underbrace{\varphi_i}_{\text{angles from } s \text{ to poles}} = 180^\circ$$

So, we want $\psi = 180^\circ + \sum_i \varphi_i$

Pole Placement Using RL



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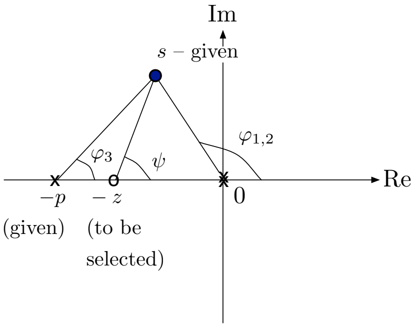

Suppose

$\varphi_1 = \varphi_2 = 120°$,

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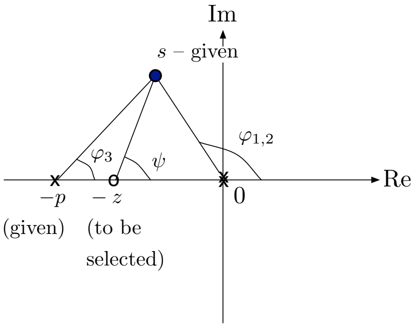

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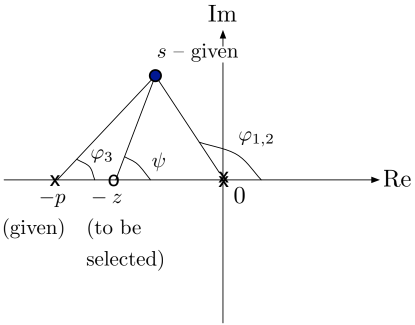

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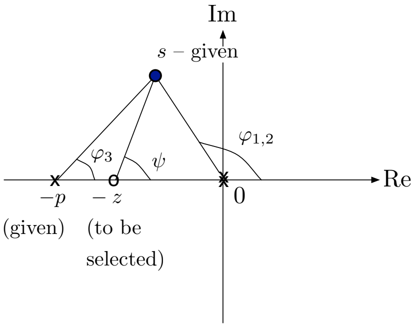

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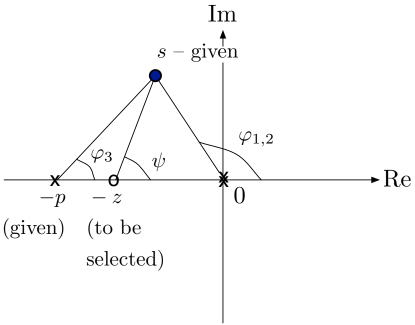

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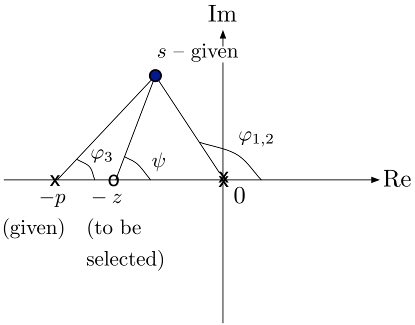

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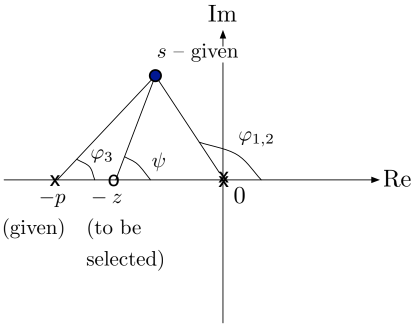

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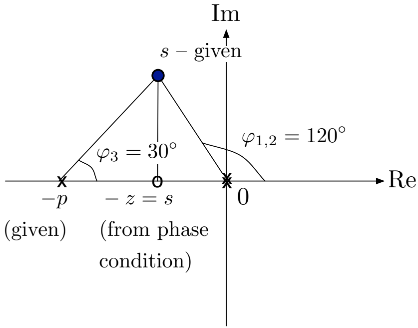

Control Design Using Root Locus

Case study: plant transfer function $G_p(s) = \dfrac{1}{s-1}$

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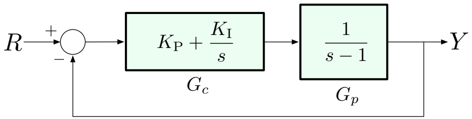

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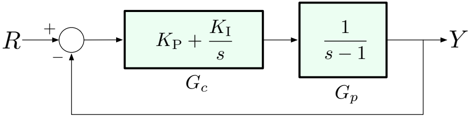
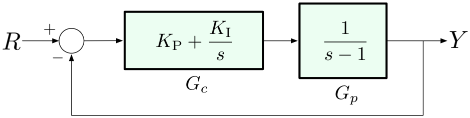
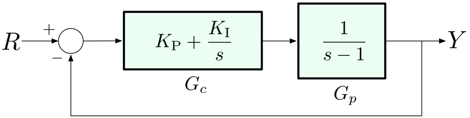


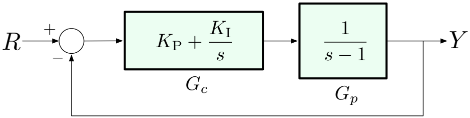
Closed-loop poles are determined by:

$$1 + \left(K_P + \frac{K_I}{s}\right)\left(\frac{1}{s-1}\right) = 0$$


R
+
−
$K_P + \frac{K_I}{s}$
G_c
$\frac{1}{s-1}$
G_p
Y



Characteristic equation: $$1 + \underbrace{\left(K_{\mathrm{P}} + \frac{K_{\mathrm{I}}}{s}\right)}_{G_c(s)} \underbrace{\left(\frac{1}{s-1}\right)}_{G_p(s)} = 0$$



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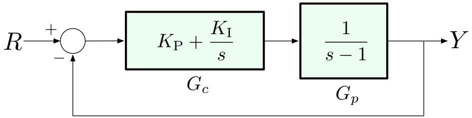
To use the RL method, we need to convert it into the Evans form $1 + KL(s) = 0$, where $L(s) = \dfrac{b(s)}{a(s)} = \dfrac{s^m + b_1 s^{m-1} + \dots}{s^n + a_1 s^{n-1} + \dots}$



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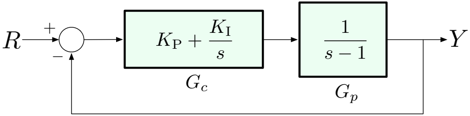

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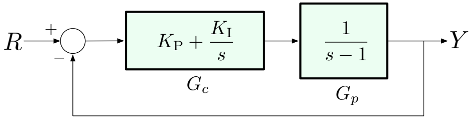

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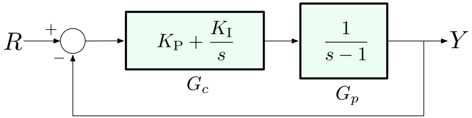

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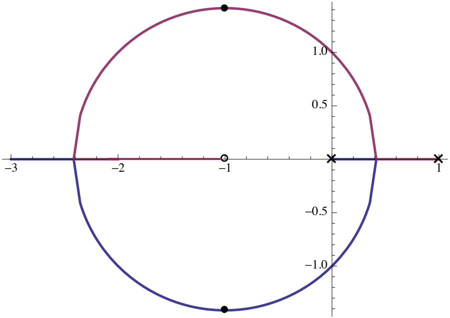

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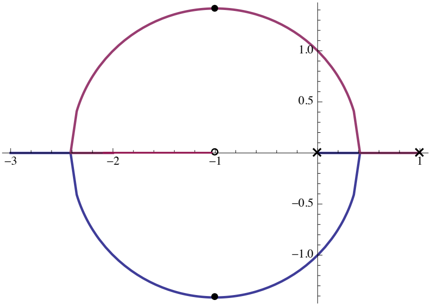
Root Locus for PI Compensation



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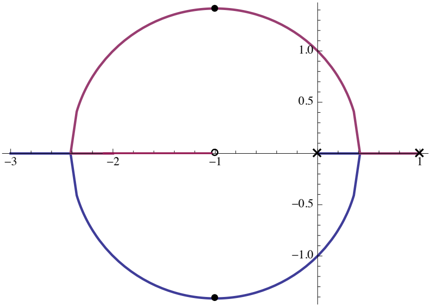

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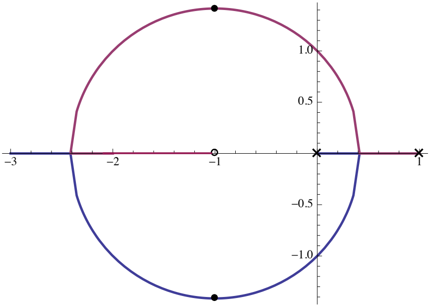

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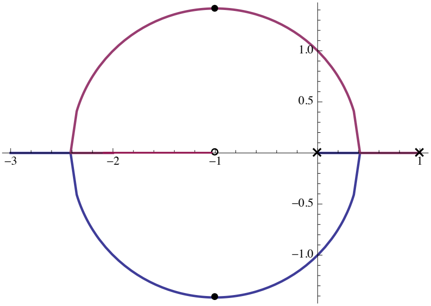

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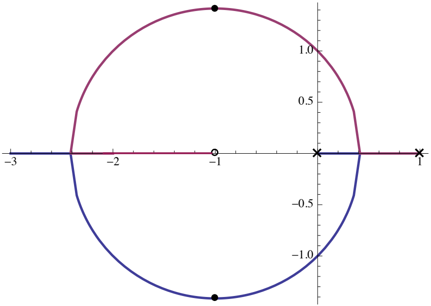

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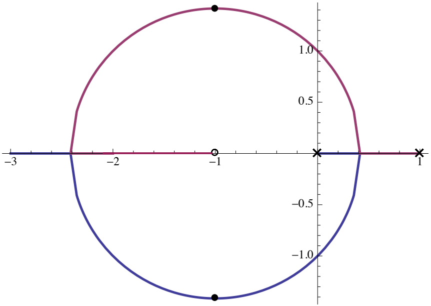
$$\frac{E}{R} = \frac{1}{1 + G_c G_p}$$

Root Locus for PI Compensation



- The system is stable for $K > 1$ (from Routh-Hurwitz)
- For very large K, we get a completely damped system, with *negative real poles*
- Perfect steady-state tracking of constant references:

$$\frac{E}{R} = \frac{1}{1 + G_c G_p} = \frac{s(s-1)}{s(s-1) + K(s+1)}$$

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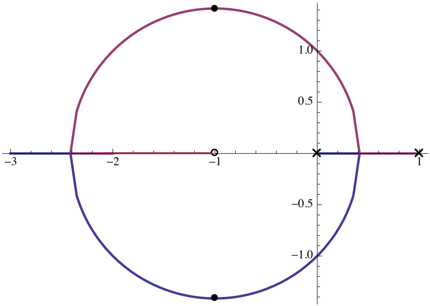

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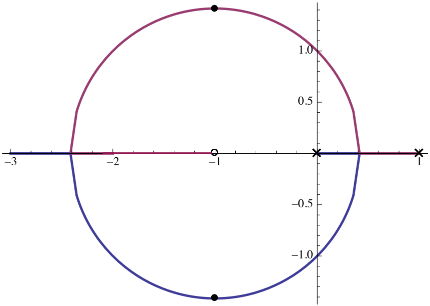

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- However: $1/s$ is not a stable element.