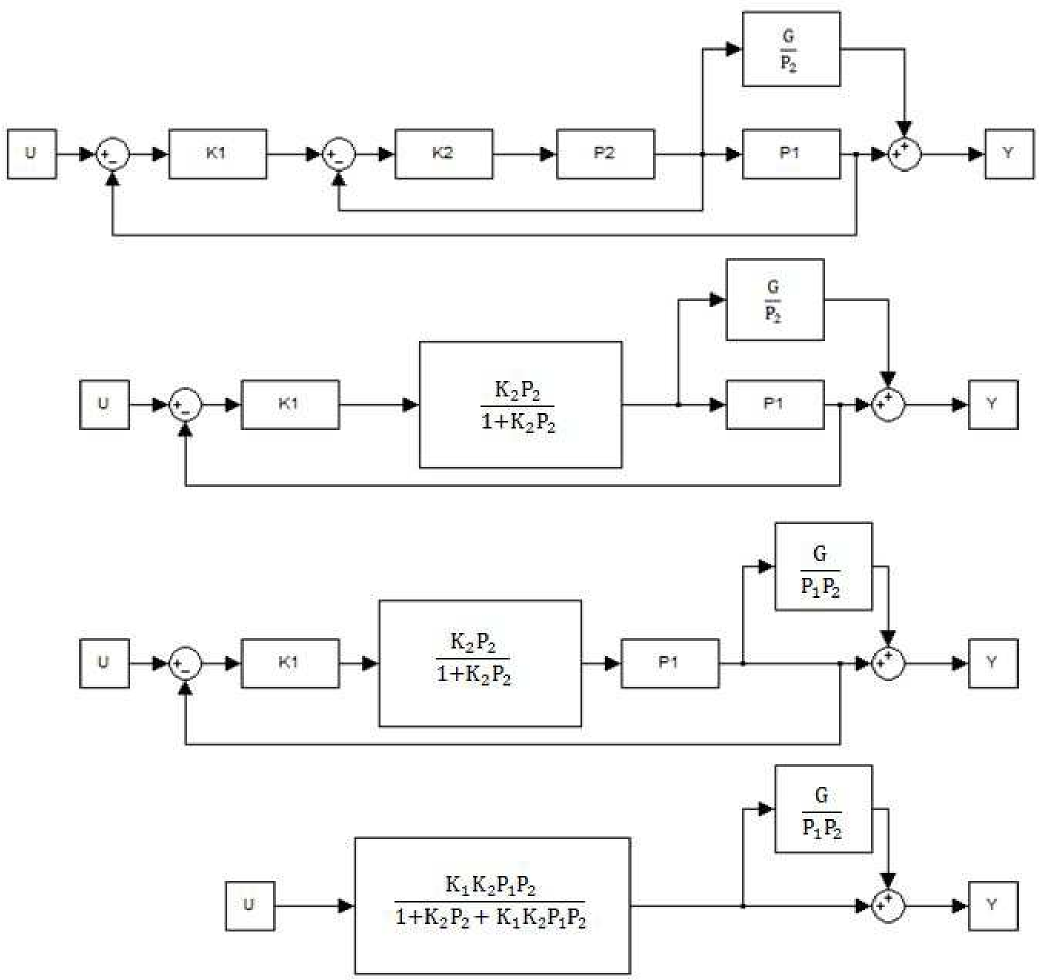
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

Department of Electrical and Computer Engineering

ECE 486: Control Systems

Sample Midterm 1 Solutions

Problem 1



$$\frac{Y}{U} = \left(1 + \frac{G}{P_1 P_2}\right)\left(\frac{K_1 K_2 P_1 P_2}{1 + K_2 P_2 + K_1 K_2 P_1 P_2}\right)$$

Problem 2

Second-order system with peak time $t_p = 0.5$ sec. and 5% settling time $t_s = 1.5$ sec.

(a) Determine the poles of this system

The poles are

$$s = -\sigma \pm j\omega_d,$$

where $\sigma = \zeta\omega_n$ and $\omega_d = \omega_n\sqrt{1-\zeta^2}$

From the problem, we are given t_p and $t_s^{5\%}$:

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d} = 0.5$$
$$\therefore \omega_d = \frac{\pi}{0.5} = 2\pi$$

$$t_s^{5\%} = \frac{3}{\zeta\omega_n} = \frac{3}{\sigma} = 1.5$$
$$\therefore \sigma = \frac{3}{1.5} = 2$$

Hence, the poles of this system are at

$$s = -2 \pm j2\pi$$

(b) Does the system satisfy the spec $t_r \leq 0.18$ sec.?

$$t_r = \frac{1.8}{\omega_n} \leq 0.18$$
$$\therefore \text{ we must have } \omega_n \geq \frac{1.8}{0.18} = 10$$

We know that $\omega_n^2 = \sigma^2 + \omega_d^2$. In our case,

$$\omega_n^2 = 2^2 + (2\pi)^2 \leq 2^2 + (2 \cdot 4)^2 = 68 < 100.$$

Therefore, the t_r spec is not satisfied.

Problem 3

Closed-loop transfer function:

$$H(s) = \frac{G_cG_p}{1 + G_cG_p}$$

Closed-loop characteristic equation:

$$1 + KL(s) = 1 + G_c G_p = 1 + K\frac{s}{(s^2-1)(s+1)}$$

Characteristic polynomial:

$$a(s) + Kb(s) = (s^2-1)(s+1) + Ks = s^3 + s^2 + (K-1)s - 1$$

Plot root locus

A: $n = 3$, $m = 1$

B: Poles at $s = 1, -1, -1$

C: Zeros at $s = 0$

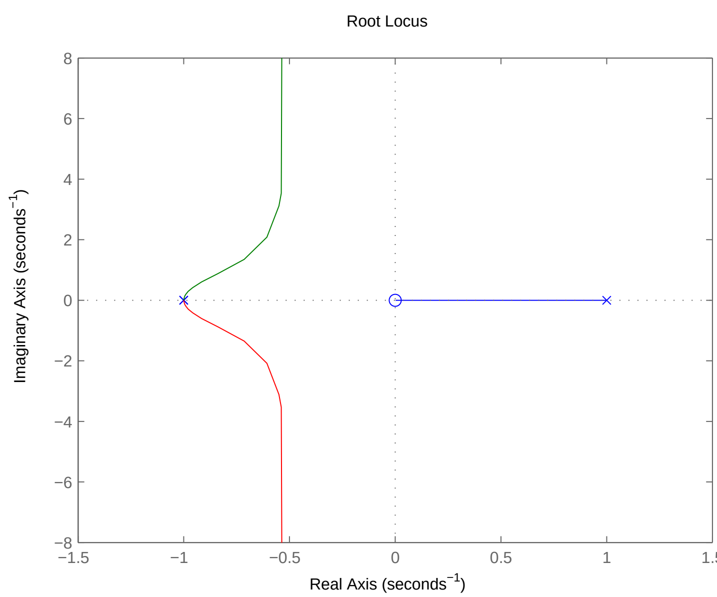
D: Real locus on (0,1)

E: $\alpha = \dfrac{\sum p - \sum z}{n-m} = -\dfrac{1}{2}$

$\phi_l = \dfrac{180° + 360° l}{n-m}$ where $l = 0, 1, ..., n-m-1$

$\therefore \phi_1 = 90°, \phi_2 = -90°$

F: no $j\omega$-crossing



This root locus has a branch in the RHP, so the system is unstable for all $K > 0$. (This can also be seen from the characteristic polynomial — it does not satisfy the necessary condition for stability.)