ECE 486: Control Systems

Lecture 9A: PI Tuning for First-Order Systems

Key Takeaways

This lecture describes a method to tune PID controllers using pole placement.

For first-order systems, the approach is to:

- Use PI control and
- Select the gains to place the two closed-loop poles at desired locations.

The choice of natural frequency (time constant) is critical.

Problem 1

Consider the plant with the following transfer function:

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

The poles of this system are at s=-1, -10±j.

A) What is the dominant pole approximation $G_a(s)$ for this plant?

B) Would you recommend using a PI, PD, or PID Controller?

C) Choose the controller gains so that the closed-loop with $G_a(s)$ has poles repeated at $s=-1$.

D) Where are the poles for the closed-loop with your controller and the actual plant $G(s)$? [Use numerical tools to solve.]

Solution 1A

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

A) What is the dominant pole approximation $G_a(s)$ for this plant?

$s = -1, \quad -10 \pm j$

slower (dominant)

$$G_a(s) = \frac{b_0}{s+1}$$

$$5 = \frac{505}{101} = G(0) = G_a(0) = b_0 \rightarrow b_0 = 5$$

$$G_a(s) = 5/(s+1)$$

$$\dot{y} + y = 5u$$

Solution 1B

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

B) Would you recommend using a $\boxed{\text{PI}}$, P~~D~~, or P~~I~~D Controller?

$G_a(s)$ first-order $\leftarrow$ Overdamped

Solution 1C

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

C) Choose the controller gains so that the closed-loop with *$G_a(s)$* has poles repeated at *s=-1*.

$(s+1)^2 = 0 \leftarrow$ Desired C.E.

$s^2 + 2s + 1 = 0$

Notes

$\boxed{\omega_n = 1 \quad \zeta = \cancel{2}\, 1}$

$s^2 + (2\zeta\omega_n)s + \omega_n^2 = 0$

$\underbrace{2\zeta\omega_n}_{2}$, $\omega_n^2 = 1$

$\omega_n = 2$

$\zeta = 0.5$

$s^2 + 2s + 4 = 0$

$G_a(s) = \frac{5}{s+1}$

$\dot{y} + y = 5u$

$u = K_p e_{(r-y)} + K_i \int e_{(r-y)}$

$\dot{y} + y = 5[K_p(r-y) + K_i \int (r-y)]$

$\downarrow d/dt$

$\ddot{y} + \dot{y} = 5K_p(\dot{r} - \dot{y}) + 5K_i(r - y)$

$\ddot{y} + [1 + 5K_p]\dot{y} + 5K_i y = 5K_p \dot{r} + 5K_i r$

$s^2 + \boxed{\underbrace{(1+5K_p)}_{=2}} s + \underbrace{\boxed{5K_i}}_{=1} = 0 \rightarrow$

$\boxed{K_i = 1/5 = 0.2 \quad K_p = 1/5 = 0.2}$

Solution 1D

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

D) Where are the poles for the closed-loop with your controller and the actual plant *G(s)*? [Use numerical tools to solve.]

$K_p = 0.2$ $K_i = 0.2$ → On G_a this put poles at $\underline{\underline{s = -1}}$

<u>Matlab</u>

G is 3rd order
K is 1st order
} Closed-loop has 4 poles

From Matlab, the four poles are at -12.643, -6.032, -1.324, -1.

Problem 2

Again consider the plant with the following transfer function:

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

The poles of this system are at s=-1, -10±j.

A) Rechoose your controller gains so that the closed-loop with $G_a(s)$ has poles repeated at ***s=-2.***

B) Where are the poles for the closed-loop with your controller and the actual plant *G(s)*? [Use numerical tools to solve.]

C) What is the impact, if any, of the neglected poles?

Solution 2A

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

A) Rechoose your controller gains so that the closed-loop with $G_a(s)$ has poles repeated at ***s=-2.***

(From Problem 1)

$\boxed{\text{Closed-loop ODE from } r \to y}$

$\ddot{y} + (1+5K_p)\dot{y} + (5K_i)y = (5K_p)\dot{r} + (5K_i)r$

$$T_{r\to y}(s) = \frac{(5K_p)s + (5K_i)}{s^2 + (1+5K_p)s + (5K_i)}$$

$T_{r\to y}(0) = \frac{5K_i}{5K_i} = 1$

If $r = \bar{r}$ then $y \to \bar{y} = \bar{r}$ (Integral Control)

$\boxed{\text{Desired C.E.}}$

$(s+2)^2 = 0 \longleftrightarrow s^2 + 4s + 4 = 0$

$s^2 + (1+5K_p)s + (5K_i)$

$K_i = 4/5 = 0.8$

$(1+5K_p) = 4 \longleftrightarrow K_p = 3/5 = 0.6$

Solution 2B and 2C

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

B) Where are the poles for the closed-loop with your controller and the actual plant *G(s)*? [Use numerical tools to solve.]

C) What is the impact, if any, of the neglected poles?

Answers: From Matlab, the four poles are at -14.42, -2.53 + 3.46i, -2.53 - 3.46i, and -1.53.

The neglected poles lead to over shoot and oscillations. We can also see that the controller is too aggressive in generating faster response (moving the pole from -1 to -2). Then the controller uses large efforts and excites the unmodeled dynamics.

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Lecture 9B: PID Tuning for Second-Order Systems

Key Takeaways

This lecture describes a method to tune PID controllers using pole placement.

For second-order systems, the approach is to:

- Use PID control and
- Select the gains to place the three closed-loop poles at desired locations.
- A PI controller (without the D-term) should be used if the plant has sufficient damping.

The choice of natural frequency (time constant) is critical.

Problem 3

Consider the plant with the following transfer function:

$$G(s) = \frac{20}{s^2 - 6s + 10}$$

A) What is the closed-loop ODE from reference *r* to output *y* if you use a PID controller? $u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \dot{e}(t)$

$-K_d \dot{y}$

B) Choose the controller gains so that the closed-loop has poles repeated at *s=-3*. Hint: *$(s+3)^3 = s^3 + 9 s^2 + 27 s + 27$*

C) What is the impact of implementing the derivative term as $K_d\ \dot{e}$ versus the rate feedback form $-K_d \dot{y}$?

Solution 3A

$$G(s) = \frac{20}{s^2 - 6s + 10}$$

A) What is the closed-loop ODE from reference *r* to output *y* if you use a PID controller? $u(t) = K_p e(t) + K_i \int_0^t e(\tau)\,d\tau + K_d \dot{e}(t)$

$\boxed{\text{Plant}}$ $\ddot{y} - 6\dot{y} + 10y = 20u$

$r - y$ $\quad -K_d\dot{y}$

$\dddot{y} - 6\ddot{y} + 10\dot{y} = 20\dot{u} = 20[K_p\dot{e} + K_i e + \boxed{K_d\ddot{e}}]$

$\dddot{y} + [20K_d - 6]\ddot{y} + [10 + 20K_p]\dot{y} + 20K_i y$

$= 20K_d\ddot{r} + 20K_p\dot{r} + 20K_i r$

$$\boxed{\underset{r\to y}{T(s)} = \frac{\cancel{20K_d s^2} + 20K_p s + 20K_i}{s^3 + [20K_d - 6]s^2 + [20K_p + 10]s + 20K_i}}$$

Desired $\rightarrow s^3 + 9s^2 + 27s + 27 = 0$

~~$K_i = 27/20$~~ $\quad K_i = 27/20 \quad K_p = \frac{(27-10)}{20} \quad K_d = \frac{(9+6)}{20}$

Solution 3B

$$G(s) = \frac{20}{s^2 - 6s + 10}$$

B) Choose the controller gains so that the closed-loop has poles repeated at *s=-3*. Hint: $0 = \boxed{(s+3)^3} = s^3 + \underline{9}\, s^2 + \underline{27}\, s + \underline{27}$

$K_d = 0.75$ $K_p = 0.85$ $K_i = 1.35$

Solution 3C

$$G(s) = \frac{20}{s^2 - 6s + 10}$$

C) What is the impact of implementing the derivative term as $K_d\ \dot{e}$ versus the rate feedback form $-K_d\dot{y}$?

$K_d(\dot{r} - \dot{y})$

Differentiating reference

Causes overshoot / oscillations if r changes fast.

$K_d\dot{e} \rightarrow$ Causes an extra term in the numerator $T_{ry}(s)$ (extra zero)