ECE 486: Control Systems

Lecture 4A: Time Domain Performance

Key Takeaways

This lecture defines important performance characteristics for a system in terms of its step response.

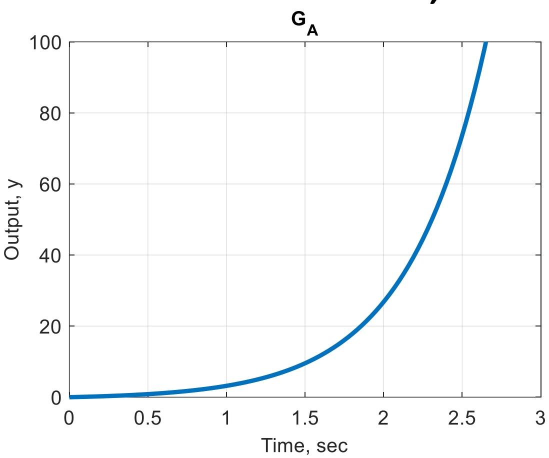
The performance characteristics include:

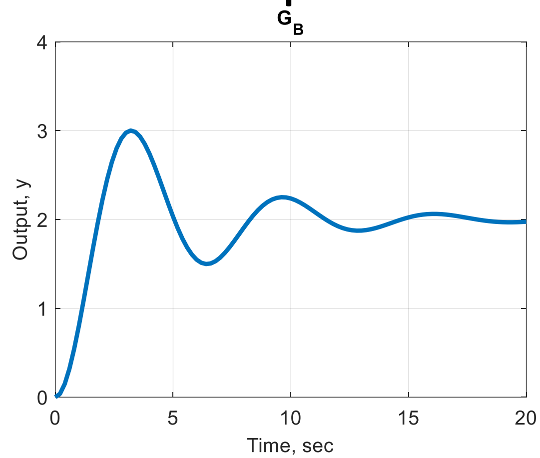
- Stability
- Final Value
- Settling Time
- Overshoot
- Rise Time
- Undershoot

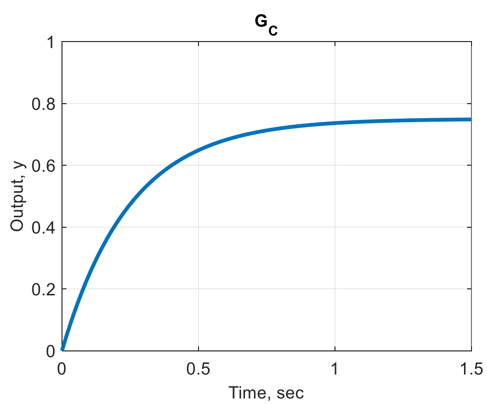
Problem 1

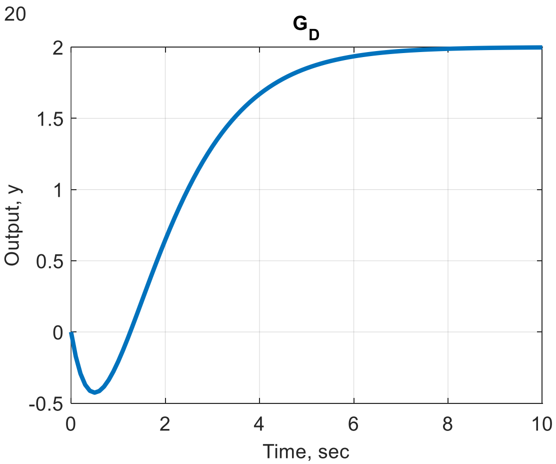
Several unit step responses are shown below. For each:

- Is the system stable?
- If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?



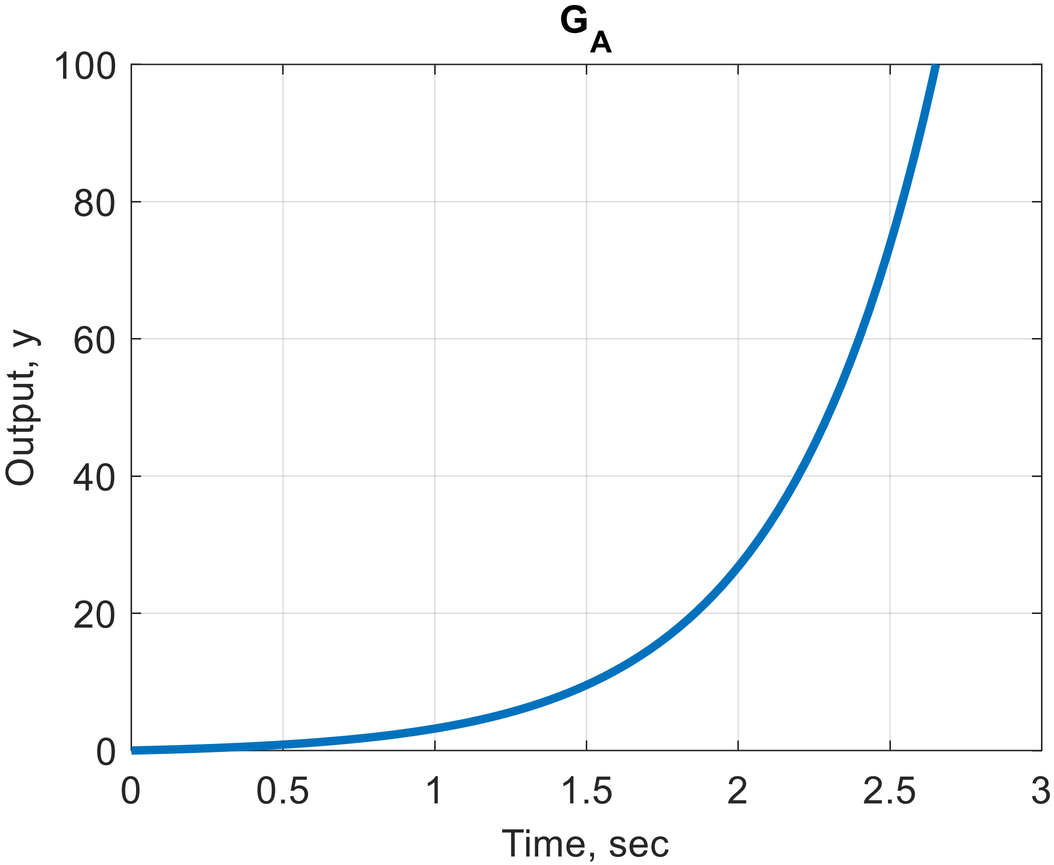






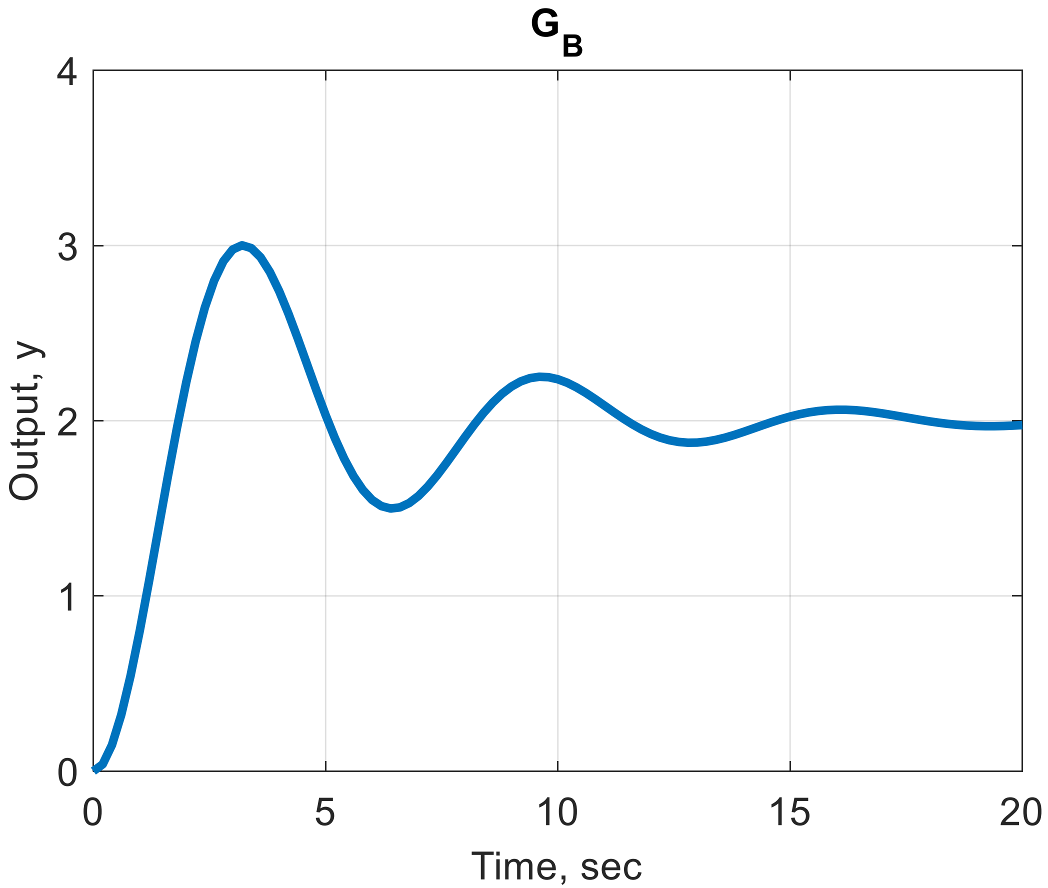
Solution 1A

- Is the system stable?
- If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?



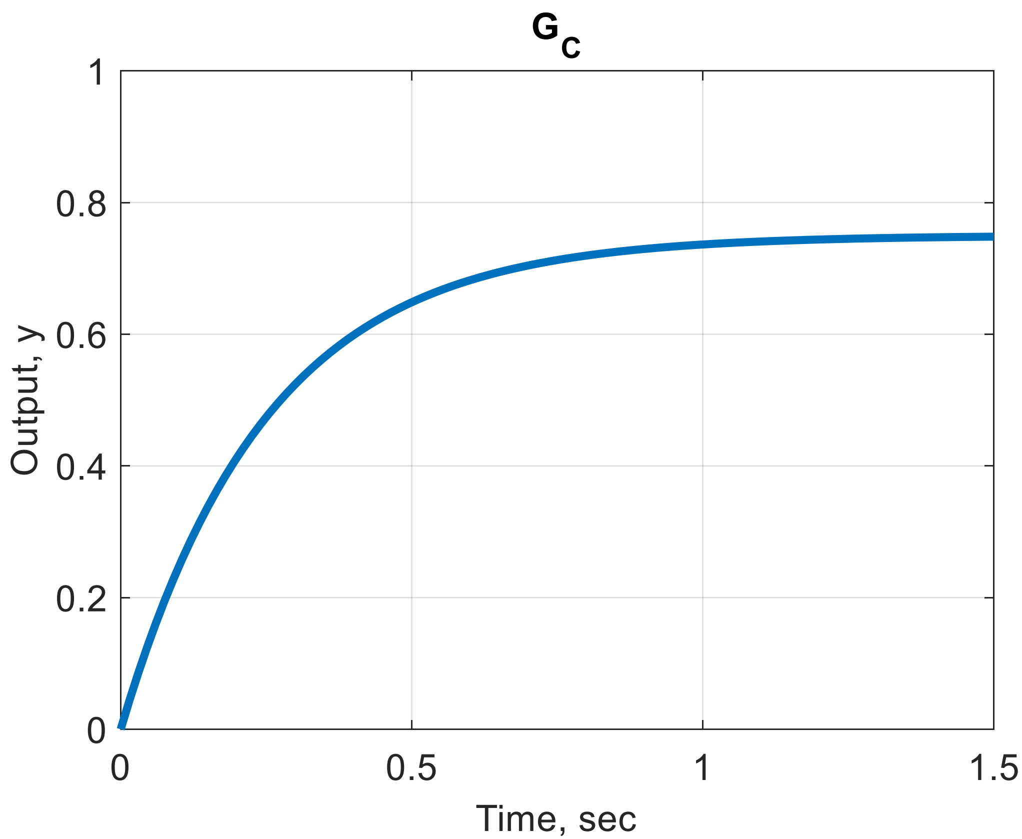
Solution 1B

- Is the system stable?
- If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?



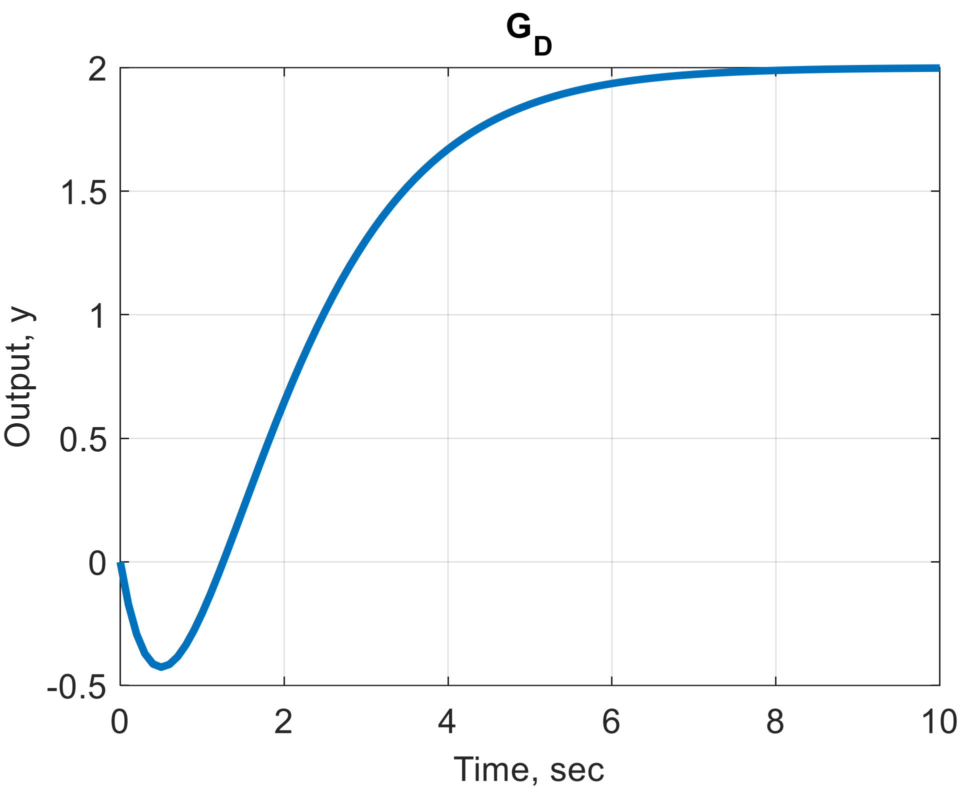
Solution 1C

- Is the system stable?
- If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?



Solution 1D

- Is the system stable?
- If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?



Solution 1-Extra Space

ECE 486: Control Systems

Lecture 4B: First-Order Step Response

Key Takeaways

This lecture covers the step response for first-order systems.

The step response of a *stable*, first-order system.

1. Converges to the final value with neither overshoot nor oscillations.
2. Has a settling time of three time constants.

Problem 2

A) Roughly sketch the response for the following:

$$\dot{y}(t) + 2y(t) = 4u(t)$$
$$\text{with } y(0) = 0 \text{ and } u(t) = 3 \text{ for all } t \geq 0$$

B) Roughly sketch the response for the following

$$\dot{y}(t) - 3y(t) = 2u(t)$$
$$\text{with } y(0) = 0 \text{ and } u(t) = 1 \text{ for all } t \geq 0$$

Solution 2A

A) Roughly sketch the response for the following:

$$\dot{y}(t) + 2y(t) = 4u(t)$$

$$\text{with } y(0) = 0 \text{ and } u(t) = 3 \text{ for all } t \geq 0$$

Solution 2B

B) Roughly sketch the response for the following

$$\dot{y}(t) - 3y(t) = 2u(t)$$

$$\text{with } y(0) = 0 \text{ and } u(t) = 1 \text{ for all } t \geq 0$$

Solution 2-Extra Space

ECE 486: Control Systems

Lecture 4C: Second-Order Step Response

Key Takeaways

This lecture covers the step response for second-order systems.

The step response of a *stable*, second-order system.

1. Is characterized by the natural frequency and damping ratio of the system
2. Has overshoot and oscillations if the system is underdamped.

Problem 3

Each of the second-order systems below is stable*

For each system:

- What is the natural frequency and damping ratio?
- Is the system under, over, or critically damped?
- Roughly sketch the unit step response noting the final time, settling time, and overshoot (if underdamped).

$$G_A(s) = \frac{20}{s^2+2s+10} \qquad G_B(s) = \frac{20}{s^2+11s+10}$$

*Recall that $s^2 + a_1 s + a_0 = 0$ has all poles in the LHP if and only if $a_1>0$ and $a_0>0$.

Solution 3A

- What is the natural frequency and damping ratio?
- Is the system under, over, or critically damped?
- Roughly sketch the unit step response noting the final time, settling time, and overshoot (if underdamped).

$$G_A(s) = \frac{20}{s^2+2s+10}$$

Solution 3B

- What is the natural frequency and damping ratio?
- Is the system under, over, or critically damped?
- Roughly sketch the unit step response noting the final time, settling time, and overshoot (if underdamped).

$$G_B(s) = \frac{20}{s^2+11s+10}$$

Solution 3-Extra Space