

ECE 486: Control Systems

Lecture 3: Response of Linear ODEs; Method of Partial Fractions; DC gain & Final Value Theorem

Key Takeaways

This lecture focuses on exact, analytical solution of the free and forced response of a linear ODE with constant coefficients.

The lecture covers the following:

1. Basic terminology: Poles, zeros, and DC (steady-state) gain
2. Minimal realizations
3. Form of the general free response solution
4. Form of the general forced response solution
5. Method of partial fractions
6. Steady state response and final value theorem (FVT)

Problem 1

Calculate the following Laplace transforms

1. $f_1(t) = 2 \cos t + \sin t$

2. $f_2(t) = e^{-2t}$

3. $f_3(t) = 2 \cos t + \sin t + e^{-2t}$

Solution 1A

Calculate the following Laplace transforms

$$f_1(t) = 2 \cos t + \sin t$$

Solution 1B

Calculate the following Laplace transforms

$$f_2(t) = e^{-2t}$$

Solution 1C

Calculate the following Laplace transforms

$$f_3(t) = 2 \cos t + \sin t + e^{-2t}$$

Solution 1 – Extra Space

Problem 2

For each of the systems below:

- What are the poles, zeros, and DC gain?
- What is the general form of the free response?
- What is the general form of the forced response?
- What is the step response with zero I.C.s?
- What is the response for zero I.C.s and $u(t)=t$ for $t \geq 0$?
- Can final value theorem be applied to determine DC gain?

If any poles are complex then express the free/forced response in its “real” form.

$$3A) G_A(s) = \frac{5}{s+7}$$

$$3B) G_B(s) = \frac{4s-6}{s^2+2s-3}$$

$$3C) G_C(s) = \frac{5}{s^2-2s+5}$$

Solution 2A

- What are the poles, zeros, and DC gain?
- What is the general form of the free response?
- What is the general form of the forced response?
- What is the step response with zero I.C.s?
- What is the response for zero I.C.s and $u(t)=t$ for $t \geq 0$?
- Can FVT be applied?

$$G_A(s) = \frac{5}{s+7}$$

Solution 2A – Extra Space

Solution 2B

- What are the poles, zeros, and DC gain? $G_B(s) = \frac{4s-6}{s^2+2s-3}$
- What is the general form of the free response?
- What is the general form of the forced response?
- What is the step response for zero I.C.s?
- What is the response for zero I.C.s and $u(t)=t$ for $t \geq 0$?
- Can FVT be applied?

Solution 2B – Extra Space

Solution 2C

- What are the poles, zeros, and DC gain?
- What is the general form of the free response?
- What is the general form of the forced response?
- What is the step response for zero I.C.s?
- What is the response for zero I.C.s and $u(t)=t$ for $t \geq 0$?
- Can FVT be applied?

$$G_C(s) = \frac{5}{s^2 - 2s + 5}$$

Solution 2C – Extra Space
