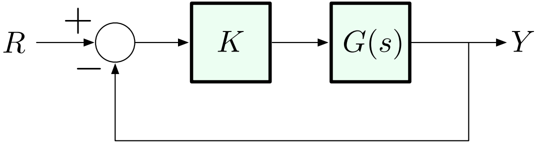
ECE 486: Control Systems

- **Lecture 19A:** Nyquist stability criterion for varying K

Goal: learn how to detect the presence of RHP poles of the closed-loop transfer function as the gain K is varied using frequency-response data

Reading: FPE, Chapter 6

Nyquist Stability Criterion



Goal: count the number of RHP poles (if any) of the closed-loop transfer function

$$\frac{KG(s)}{1 + KG(s)}$$

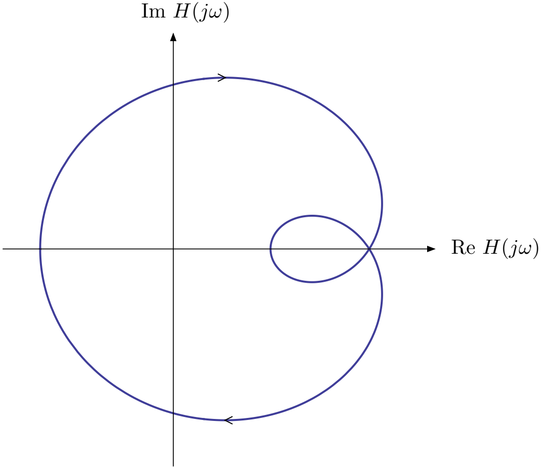
based on frequency-domain characteristics of the plant transfer function $G(s)$

Review: Nyquist Plot

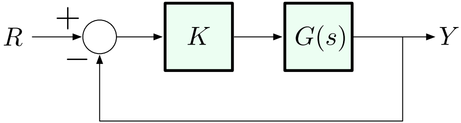
Consider an arbitrary *strictly proper* transfer function H:

$$H(s) = \frac{(s - z_1)\dots(s - z_m)}{(s - p_1)\dots(s - p_n)}, \qquad m < n$$

Nyquist plot: $\mathrm{Im}\, H(j\omega)$ vs. $\mathrm{Re}\, H(j\omega)$ as ω varies from $-\infty$ to ∞



The Nyquist Stability Criterion



$$\underbrace{N}_{\#(\circlearrowright \text{ of } -1/K)} = \underbrace{Z}_{\#(\text{unstable CL poles})} - \underbrace{P}_{\#(\text{unstable OL poles})}$$

$$Z = N + P$$

$$Z = 0 \qquad \Longrightarrow N = -P$$

Nyquist Stability Criterion. Under the assumptions of the Nyquist theorem, the closed-loop system (at a given gain K) is stable *if and only if* the Nyquist plot of $G(s)$ encircles the point $-1/K$ P times *counterclockwise*, where P is the number of unstable (RHP) open-loop poles of $G(s)$.

Applying the Nyquist Criterion

Workflow:

$$\text{Bode } M \text{ and } \phi\text{-plots} \quad \longrightarrow \quad \text{Nyquist plot}$$

Advantages of Nyquist over Routh–Hurwitz

- can work directly with experimental frequency response data (e.g., if we have the Bode plot based on measurements, but do not know the transfer function)
- less computational, more geometric (came 55 years after Routh)

Example

$$G(s) = \frac{1}{(s+1)(s+2)} \qquad \text{(no open-loop RHP poles)}$$

Characteristic equation:

$$(s+1)(s+2) + K = 0 \qquad \Longleftrightarrow \qquad s^2 + 3s + K + 2 = 0$$

From Routh, we already know that the closed-loop system is stable for $K > -2$.

We will now reproduce this answer using the Nyquist criterion.

Example

$$G(s) = \frac{1}{(s+1)(s+2)} \qquad \text{(no open-loop RHP poles)}$$

Strategy:

- Start with the Bode plot of G
- Use the Bode plot to graph $\text{Im } G(j\omega)$ vs. $\text{Re } G(j\omega)$ for $0 \le \omega < \infty$
- This gives only a *portion* of the entire Nyquist plot

$$\left(\text{Re } G(j\omega), \text{Im } G(j\omega)\right), \qquad -\infty < \omega < \infty$$

- Symmetry:

$$G(-j\omega) = \overline{G(j\omega)}$$

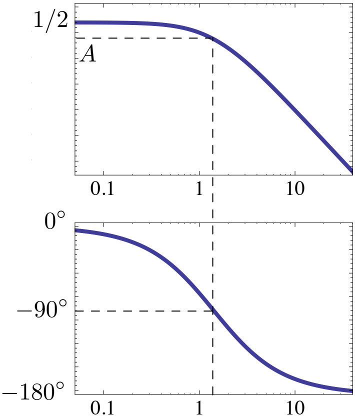
— Nyquist plots are always *symmetric w.r.t. the real axis*!!

Example

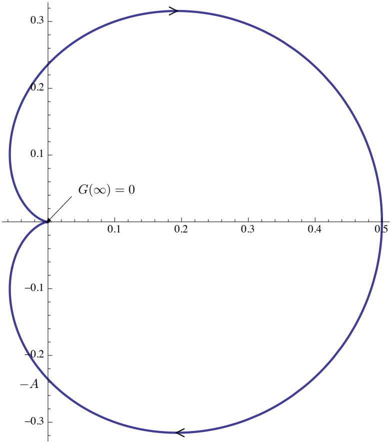
$$G(s) = \frac{1}{(s+1)(s+2)}$$

(no open-loop RHP poles)

Bode plot:



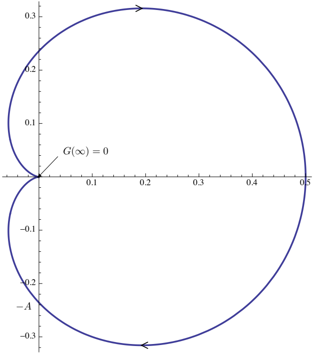
Nyquist plot:



Example: Applying the Nyquist Criterion

$$G(s) = \frac{1}{(s+1)(s+2)} \qquad \text{(no open-loop RHP poles)}$$

Nyquist plot:



$$\#(\circlearrowright \text{ of } -1/K) = \#(\text{RHP CL poles}) - \underbrace{\#(\text{RHP OL poles})}_{=0}$$

$\implies K \in \mathbb{R}$ is stabilizing if and only if

$$\#(\circlearrowright \text{ of } -1/K) = 0$$

- If $K > 0$, $\#(\circlearrowright \text{ of } -1/K) = 0$
- If $0 < -1/K < 1/2$, $\#(\circlearrowright \text{ of } -1/K) > 0 \implies$ closed-loop stable for $K > -2$