ECE 486: Control Systems

Lecture 18B: Cauchy’s Argument Principle

Key Takeaways

This lecture presents a result known as Cauchy's Argument Principle for a transfer function *G(s)*.

To state the principle:

- Let Γ be a simple, closed curve in the complex plane.
- Let N_p and N_z denote the number of poles and zeros of G(s) that lie inside the curve Γ.

Cauchy's Argument Principle: *G(s)* is evaluated on the curve Γ will encircle the origin (Nz – Np) times.

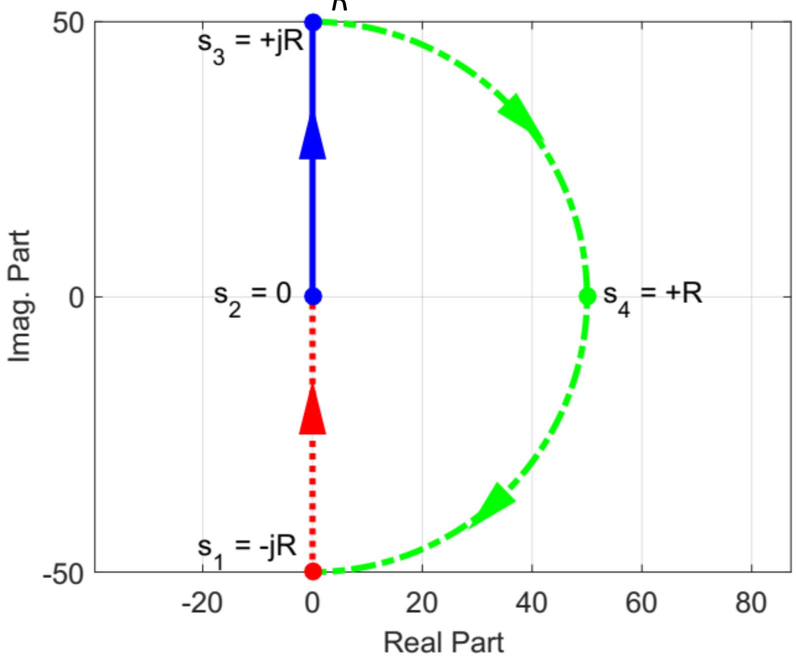
This result is used to state a theorem to assess stability of a feedback system using Nyquist plots.

Notation

Let Γ be a simple, closed curve in the complex plane:

- Simple: The curve does not intersect itself
- Closed: End point of the curve is the same as the starting point.

Γ_R with $R=50$



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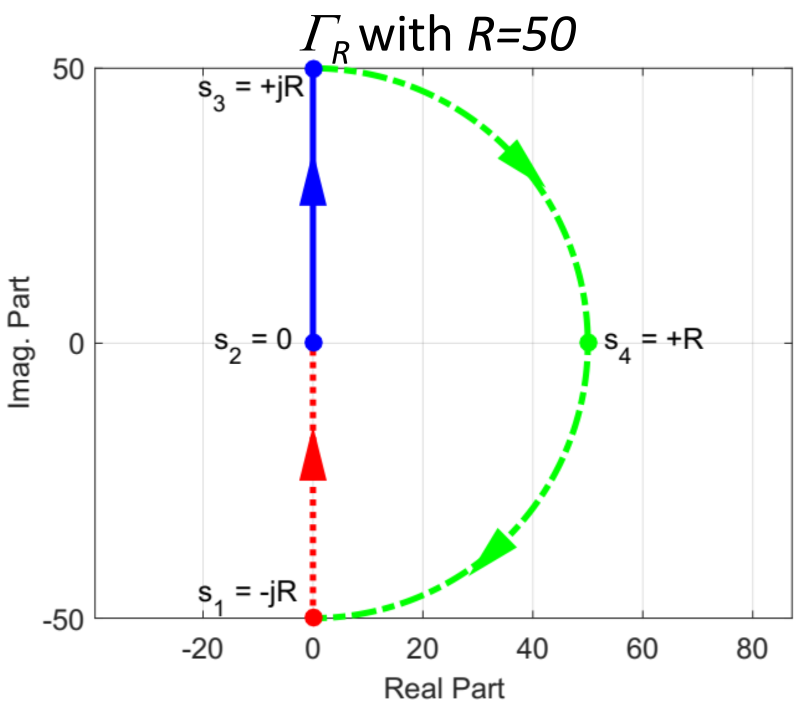
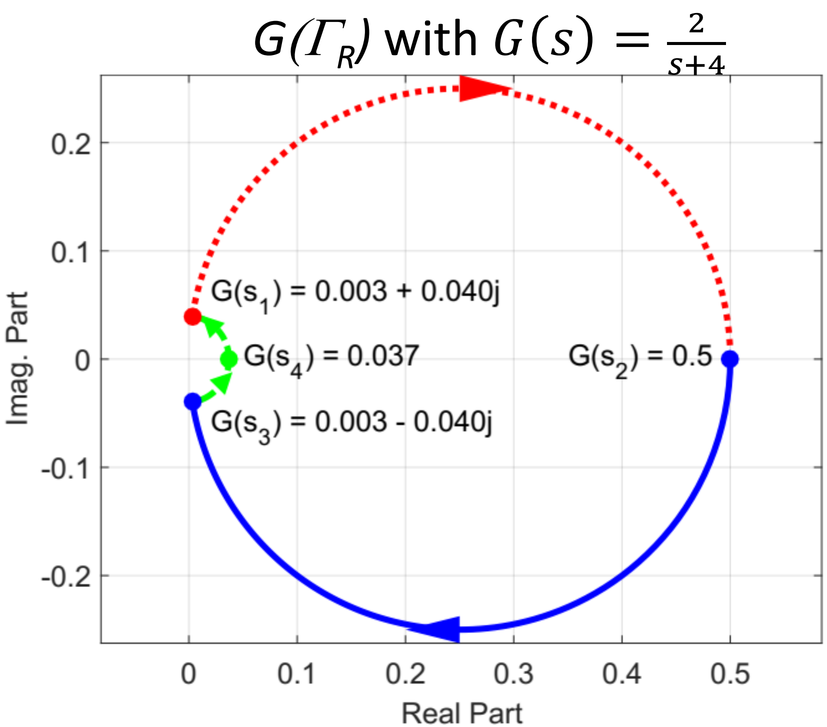
G(Γ) denotes the curve obtained by mapping each complex number $s_0 \in \Gamma$ to another complex number $G(s_0)$.





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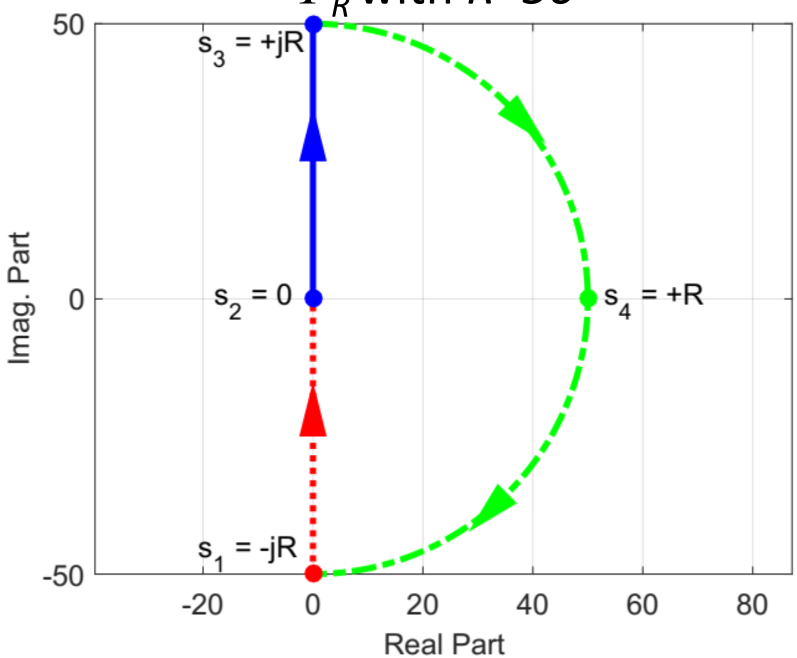
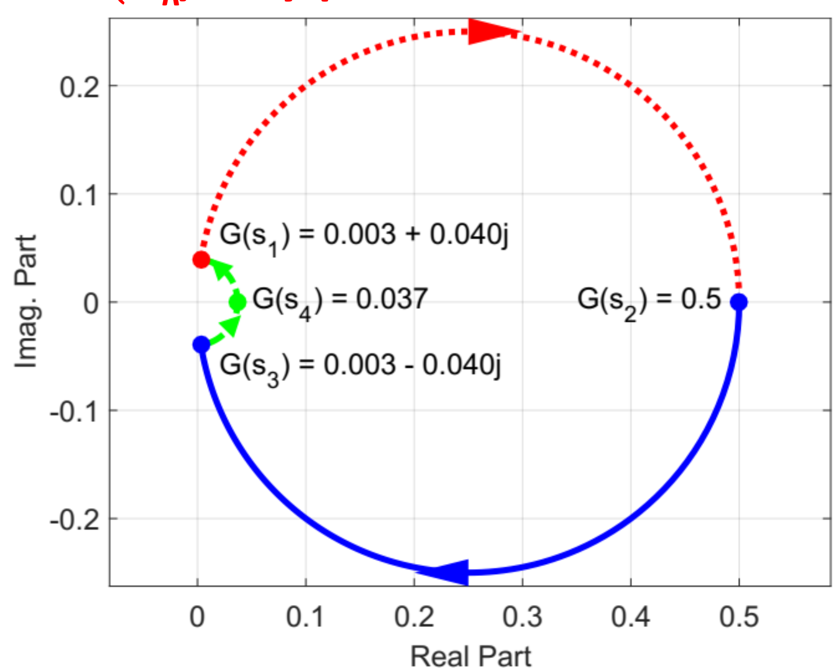
Γ_R with R=50



$G(\Gamma_R) \rightarrow$ Nyquist Plot of G as $R \rightarrow \infty$



Cauchy's Argument Principle

Define:

- N_p:=Number of poles of *G(s)* inside the curve Γ.
- N_z:=Number of zeros of *G(s)* inside the curve Γ.

Principle: Assume Γ does not pass through any poles or zeros of *G(s)*. Then:

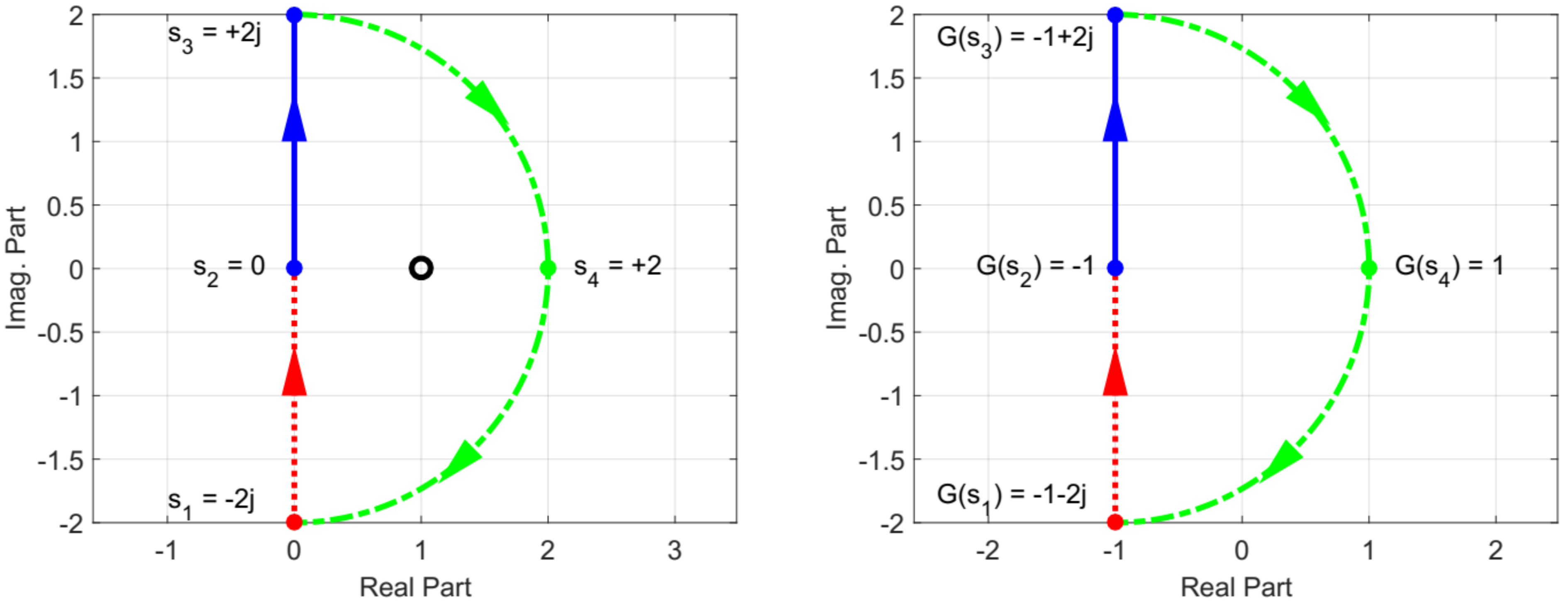
- The closed curve $G(\Gamma)$ encircles the origin Nz −Np times.
- If $N_z - N_p > 0$ then $G(\Gamma)$ encircles the origin clockwise (CW).
- If $N_z - N_p < 0$ then $G(\Gamma)$ encircles the origin counter-clockwise (CCW).

Example 1

G(s)=s-1 shifts Γ_R to the left by one unit.

- N_p:=Number of poles of *G(s)* inside the curve Γ = 0
- N_z:=Number of zeros of *G(s)* inside the curve Γ = 1

→ $G(\Gamma_R)$ encircles the origin $N_z - N_p = 1 > 0$ times CW.

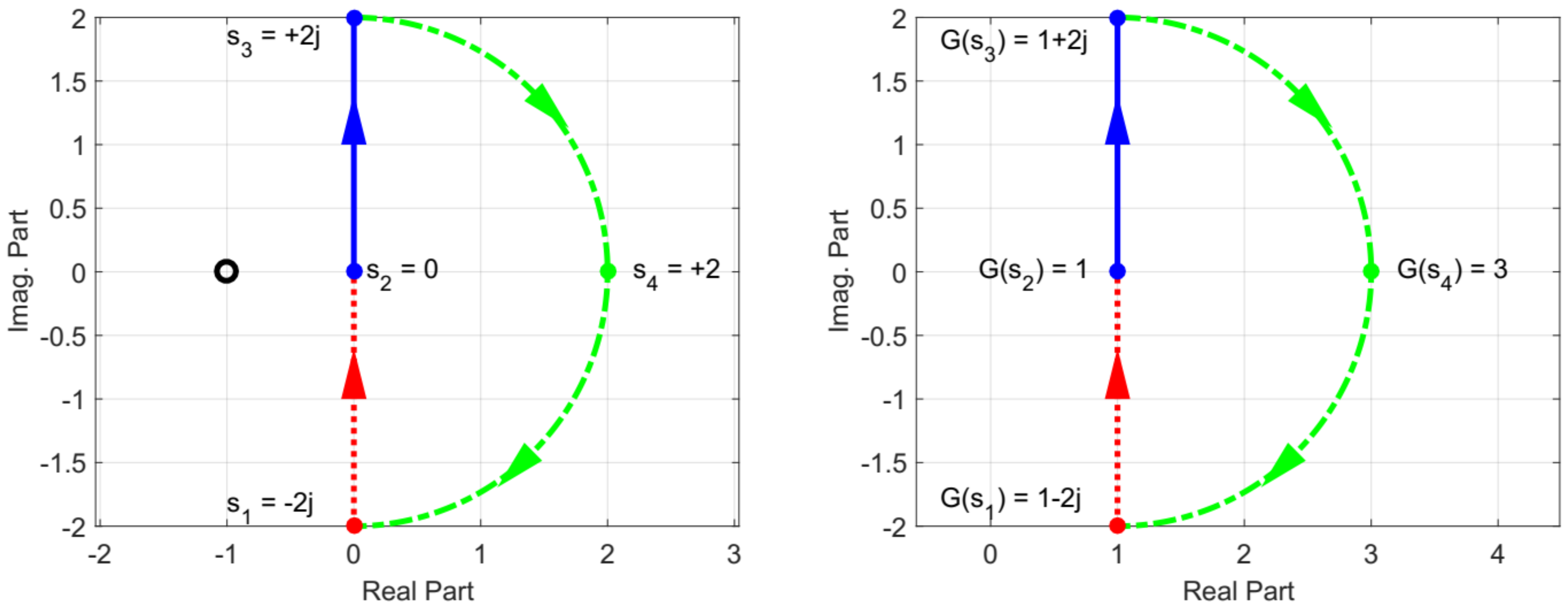


Example 2

G(s)=s+1 shifts Γ_R to the right by one unit.

- N_p:=Number of poles of *G(s)* inside the curve Γ = 0
- N_z:=Number of zeros of *G(s)* inside the curve Γ = 0

→ $G(\Gamma_R)$ encircles the origin $N_z - N_p = 0$ times.

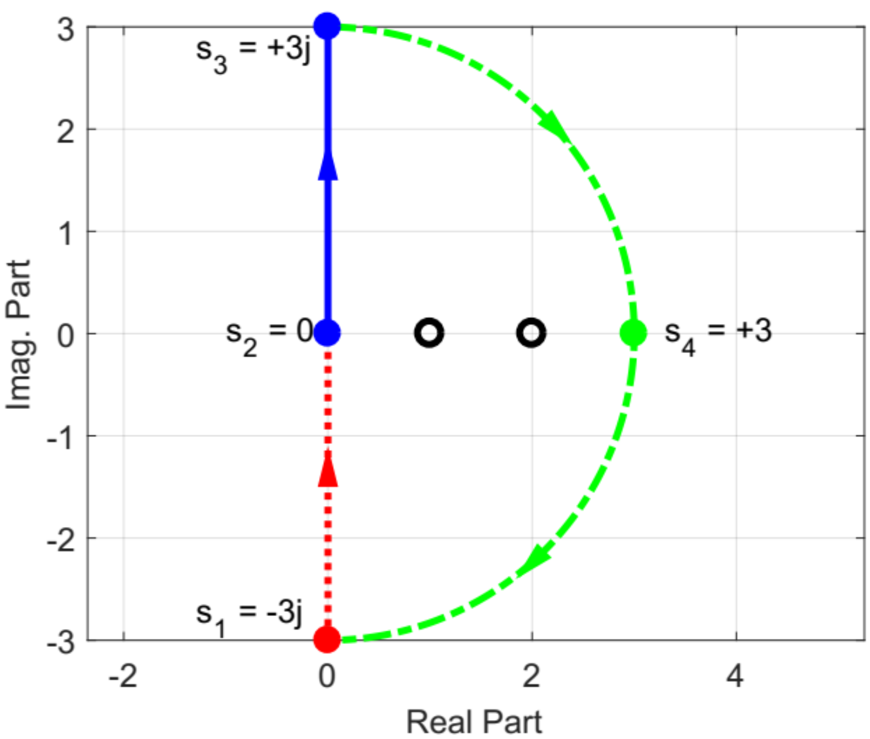


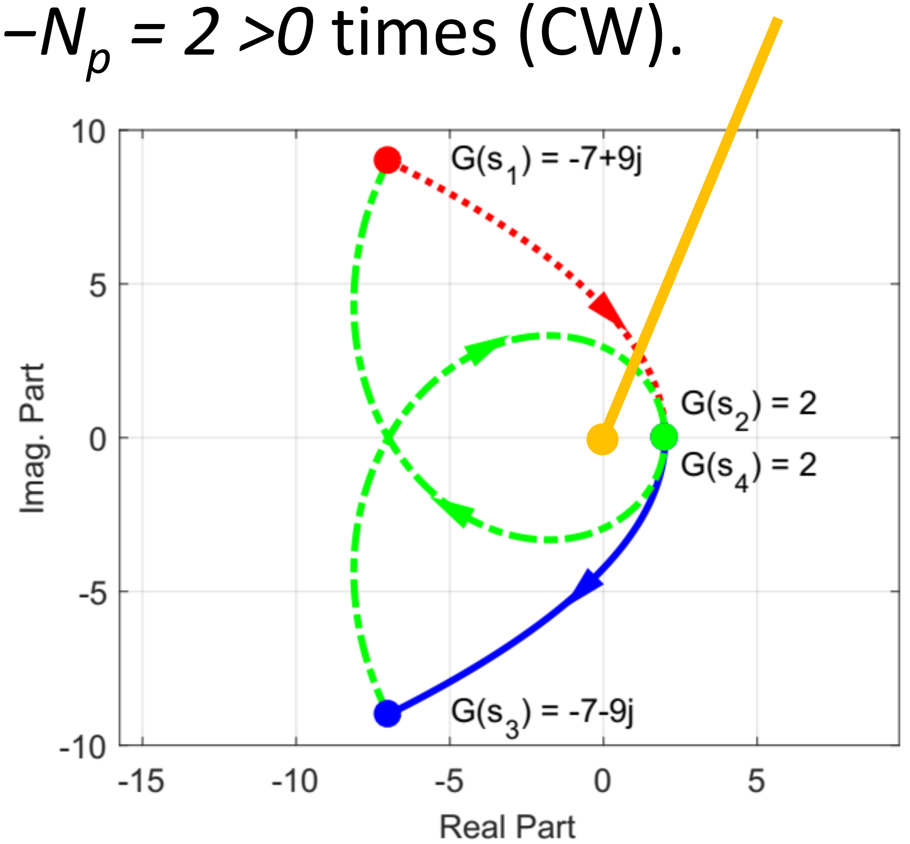
Example 3

$G(s)=s^2-3s+2$ evaluated on Γ_R is a more complicated curve.

- N_p:=Number of poles of $G(s)$ inside the curve Γ = 0
- N_z:=Number of zeros of $G(s)$ inside the curve Γ = 2

→ $G(\Gamma_R)$ encircles the origin $N_z - N_p = 2 > 0$ times (CW).





Example 4

$G(s) = \frac{2s+4}{s-1}$ evaluated on Γ_R is a more complicated curve.

- N_p:=Number of poles of *G(s)* inside the curve Γ = 1
- N_z:=Number of zeros of *G(s)* inside the curve Γ = 0

$\rightarrow G(\Gamma_R)$ encircles the origin $N_z - N_p = -1 < 0$ times (CCW).