ECE 486: Control Systems

Lecture 18A: Nyquist Plots

Key Takeaways

A Nyquist plot is a single plot of the frequency response $G(j\omega)$.

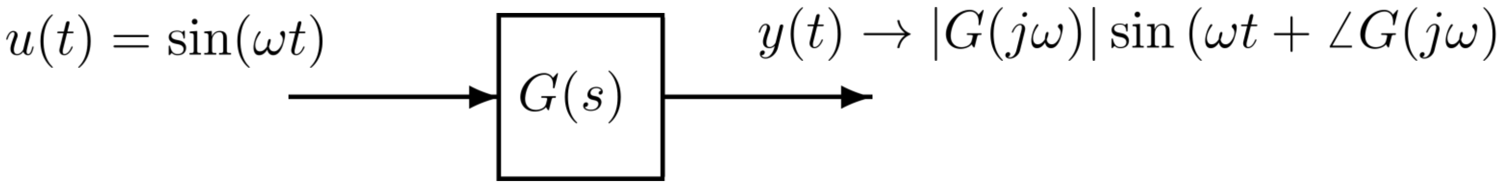
- It consists of the imaginary part $Im(G(j\omega))$ on the vertical axis versus the real part $Re(G(j\omega))$ on the horizontal axis.
- The convention is to draw this plot for both $\omega \geq 0$ and $\omega < 0$.

Nyquist plots are used to understand the stability and robustness of a feedback system.

Nyquist plots can be drawn in Matlab using the `nyquist` command. The plots for first order systems (with or without a zero) are simply circles in the complex plane.

Nyquist Plots

Recall that the steady-state sinusoidal response of a stable LTI system is determined by the magnitude and phase of G(jω).



A Bode plot displays $|G(j\omega)|$ and $\angle G(j\omega)$ versus ω on two separate plots.

A Nyquist plot displays the response G(jω) in a different form:

- A single plot of the imaginary part Im(G(jω)) vs. Re(G(jω)).
- The frequency ω is implicit on the plot.
- The convention is to draw the plot for both $\omega \geq 0$ and $\omega < 0$. These parts of the curve are complex conjugates.

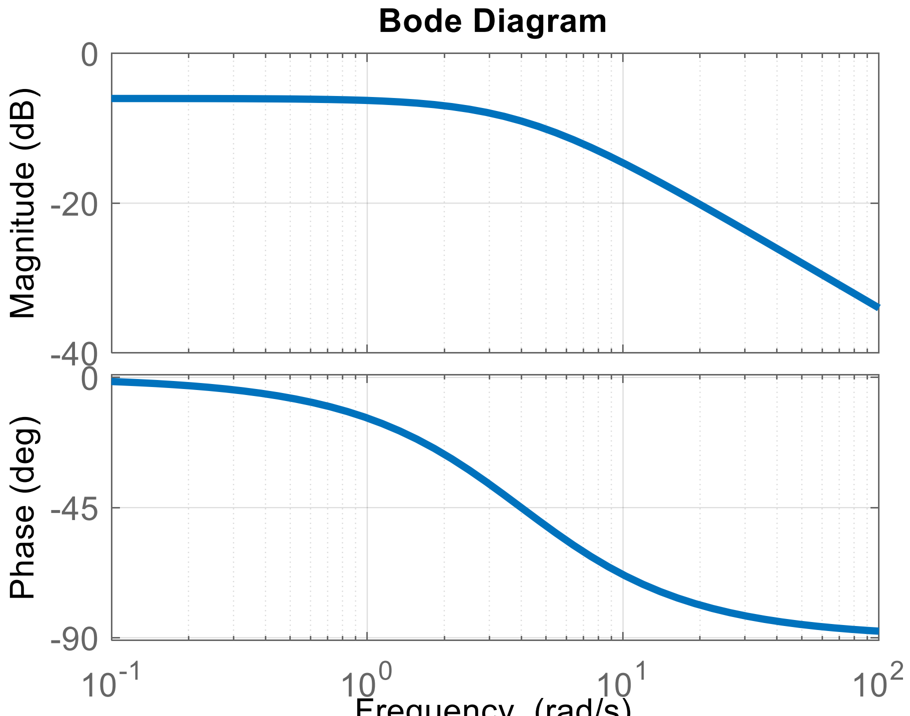
The Matlab command `nyquist` can be used to draw these plots.

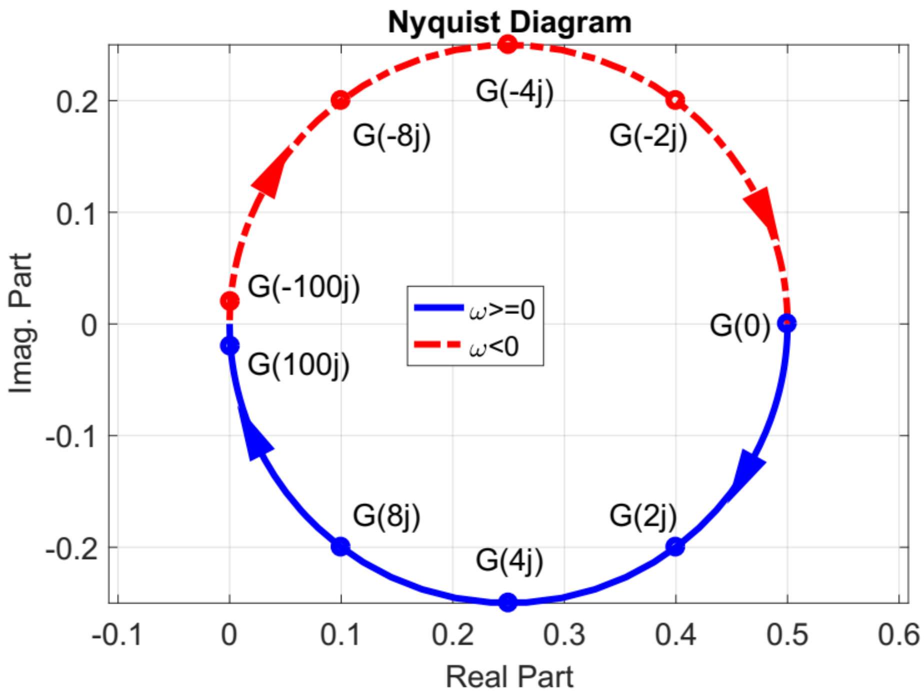
Example

Consider the stable, first-order system:

$$\dot{y}(t) + 4y(t) = 2u(t) \qquad G(s) = \frac{2}{s+4}$$

```
>> G = tf(2,[1 4]);
>> bode(G);
>> nyquist(G);
```





Nyquist Plots: First-Order Systems

Consider the stable, first-order system:

$$\dot{y}(t) + a_0 y(t) = b_0 u(t) \qquad G(s) = \frac{b_0}{s + a_0}$$

The real and imaginary parts of the frequency response are:

$$G(j\omega) = \frac{b_0}{j\omega + a_0} \cdot \frac{-j\omega + a_0}{-j\omega + a_0} = \underbrace{\frac{b_0 a_0}{a_0^2 + \omega^2}}_{Re(G(j\omega))} + j \underbrace{\frac{-b_0 \omega}{a_0^2 + \omega^2}}_{Im(G(j\omega))}$$

After some algebra, the real and imaginary parts satisfy:

$$\left(Re(G(j\omega)) - \frac{b_0}{2a_0} \right)^2 + Im(G(j\omega))^2 = \left(\frac{b_0}{2a_0} \right)^2$$

This is a circle in the complex plane with center on the real axis at $\frac{b_0}{2a_0}$ and radius $\frac{b_0}{2a_0}$.

The Nyquist plot of $G(s) = \frac{b_1 s + b_0}{s + a_0}$ is also a circle.

Example

Consider the stable, first-order system:

$$\dot{y}(t) - 4y(t) = 2u(t) \qquad G(s) = \frac{2}{s-4}$$

Sketch plot from three points:

- DC Gain:

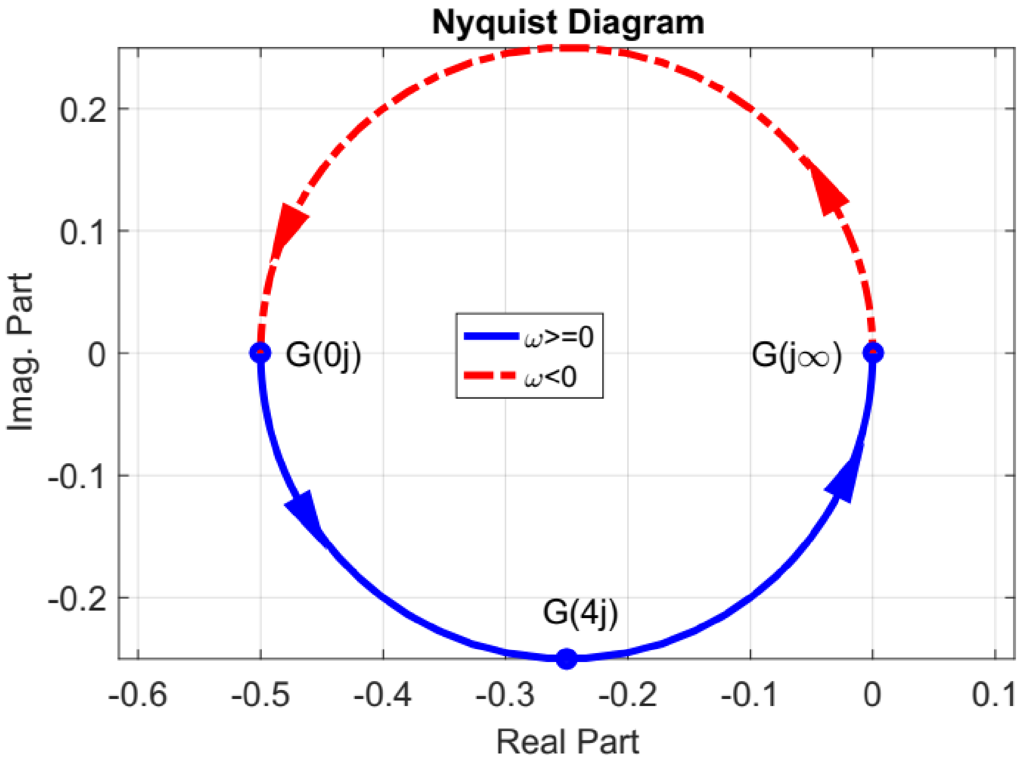
$$G(0) = -0.5$$

- High Frequency: $\omega \to \infty$

$$G(\omega) \to \frac{2}{j\omega} = -\frac{2j}{\omega}$$

- Corner Frequency: $\omega = 4\frac{rad}{sec}$

$$G(4j) \to \frac{2}{4j-4} = -0.25 - 0.25j$$



Example

Consider the stable, first-order system:

$$\dot{y}(t) - 2y(t) = 3\dot{u}(t) + 5u(t) \qquad G(s) = \frac{3s+5}{s-2}$$

Sketch plot from three points:

- DC Gain:

$$G(0) = -2.5$$

- High Frequency: $\omega \to \infty$

$$G(\omega) \to 3$$

- Corner Frequency: $\omega = 2\frac{rad}{sec}$

$$G(2j) \to \frac{6j+5}{2j-2} = 0.25 - 2.75j$$