Reading: FPE, Sections 6.1.

Problems:

1. Consider the transfer function $G(s) = \dfrac{1}{s^2 + 0.5s + 1}$.

a) Use the formulas given in class (taken from the book by Kuo, Section 9.2) to compute the resonant frequency ω_r, resonant peak M_r, and bandwidth ω_{BW} for $G(j\omega)$.

b) Use a computer to plot the magnitude $|G(j\omega)|$ as a function of ω (you can use the `bode` or `bodemag` command in MATLAB). Mark the resonant frequency ω_r, resonant peak M_r, and bandwidth ω_{BW} on the graph. Check agreement with the values you computed in a).

Solution:

a)

$$G(s) = \frac{1}{s^2 + 0.5s + 1} = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \Rightarrow \omega_n = 1, \zeta = 0.25$$

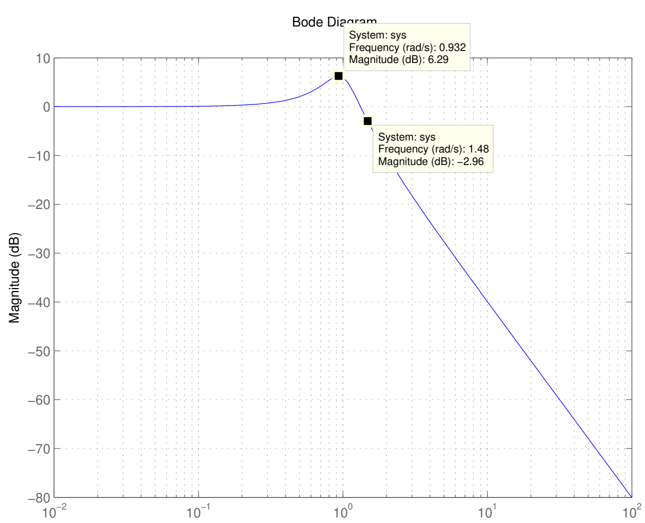
$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2} = 1 \times \sqrt{1 - \frac{1}{8}} = 0.9354$$

$$M_r = \sqrt{\left.\frac{1}{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + 4\zeta^2\left(\frac{\omega^2}{\omega_n^2}\right)}\right|_{\omega=\omega_r}} - 1 = \sqrt{\frac{1}{(2\zeta^2)^2 + 4\zeta^2(1 - 2\zeta^2)}} - 1$$

$$= \sqrt{\frac{1}{\left(1 - \frac{7}{8}\right)^2 + 4\frac{1}{16}\frac{7}{8}}} - 1 = \sqrt{\frac{64}{15}} - 1 = 1.065$$

$$\omega_{BW} = \omega_n\sqrt{1 - 2\zeta^2 + \sqrt{2 + 4\zeta^4 - 4\zeta^2}} = \sqrt{1 - \frac{1}{8} + \sqrt{2 + \frac{1}{64} - \frac{1}{4}}} = 1.4845$$

b)



2. For each of the transfer functions given below, draw the Bode plots (both magnitude and phase) *by hand*, using the techniques discussed in class. Explain all steps in your drawing procedures. Note that the transfer functions are not given in Bode form.

a) $KG(s) = \dfrac{s+10}{s(s+5)}$ b) $KG(s) = \dfrac{8s}{s^2+0.2s+4}$ c) $KG(s) = \dfrac{s^2+0.1s+1}{s(s+0.2)(s+4)}$

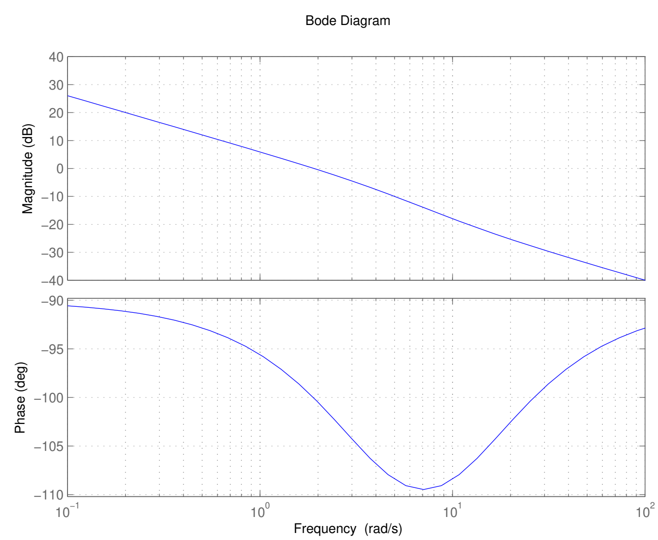
After you're done, check your results using MATLAB. (Note that the `bode` command in MATLAB plots magnitude in decibels.) Turn in both the hand sketches and the MATLAB plots.

Solution:

a)

$$\text{Bode form: } KG(s) = 2\frac{\frac{s}{10}+1}{s\left(\frac{s}{5}+1\right)}$$

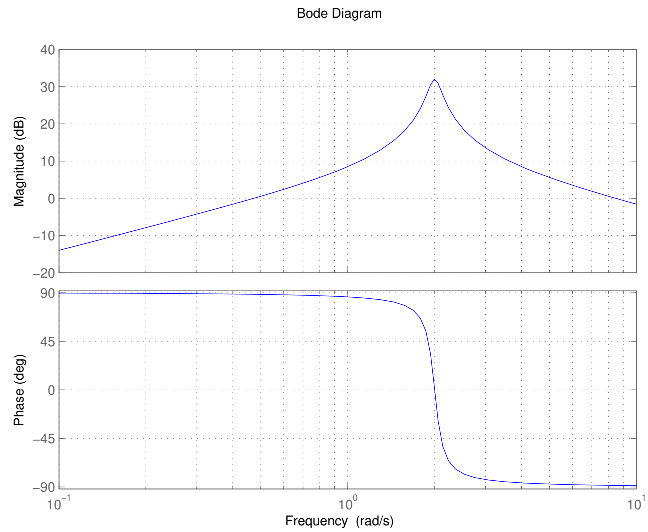
Break points: $\omega = 0$, $\omega = 5$, $\omega = 10$
$|KG(j1)| = 2$
Slope: -1 → -2 → -1
Phase: $-90° \to -180° \to -90°$



b)

$$\text{Bode form: } KG(s) = 2\frac{s}{\left(\frac{s}{2}\right)^2+0.05s+1}$$

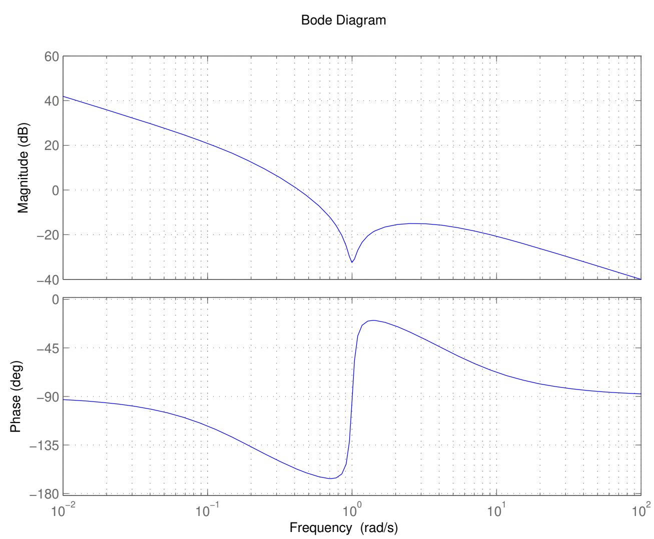
Break points: $\omega = 0$, $\omega = 2$
$|KG(j0.01)| = 0.02$
Slope: +1 → -1
Phase: $+90° \to -90°$



c)

$$\text{Bode form: } KG(s) = \frac{1}{0.8}\frac{s^2+0.1s+1}{s\left(\frac{s}{0.2}+1\right)\left(\frac{s}{4}+1\right)}$$

Break points: $\omega = 0.2$, $\omega = 1$, $\omega = 4$
$|KG(j0.01)| = 125$
Slope: -1 $\rightarrow$ -2 $\rightarrow$ 0 $\rightarrow$ -1
Phase: $-90^\circ \rightarrow -180^\circ \rightarrow 0^\circ \rightarrow -90^\circ$



3. Consider the transfer function

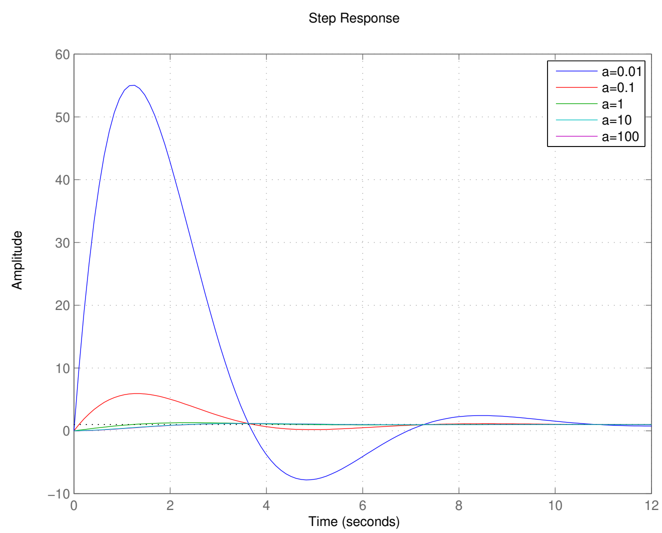
$$G(s) = \frac{\frac{s}{a} + 1}{s^2 + s + 1}$$

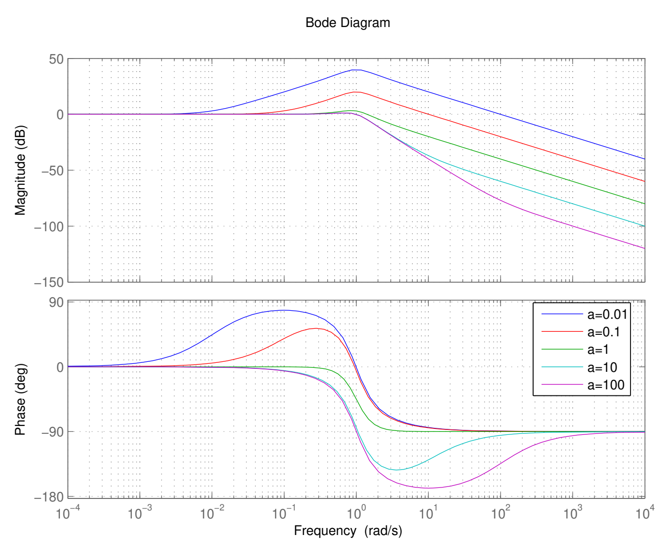
Use MATLAB to compare the M_p from the step response of the system for $a = 0.01$, 0.1, 1, 10, and 100 with the M_r from the frequency response for the same values of a. Is there a correlation between M_p and M_r?

Solution:

α	Resonant peak (M_r)	Overshoot (M_p)
0.01	98.8	54.1
0.1	9.93	4.94
1	1.46	0.30
10	1.16	0.16
100	1.15	0.16

As a is reduced, the resonant peak in frequency response increases. This leads us to expect extra peak overshoot in transient response. This effect is significant in case of $a = 0.01, 0.1, 1$, while the resonant peak in frequency response is hardly changed in case of $a = 10$. Thus, we do not have considerable change in peak overshoot in transient response for $a \geq 10$. The response peak in frequency response and the peak overshoot in transient response are correlated.





4. Consider the transfer function

$$G(s) = \frac{1}{\left(\frac{s}{p}+1\right)(s^2+s+1)}$$

Draw the Bode plots for $p = 0.01$, 0.1, 1, 10, and 100. What conclusions can you draw about the effect of the pole at $-p$ on the bandwidth compared with the bandwidth for the second-order system without this pole? MATLAB use is allowed.

Solution:

p	Additional pole $(-p)$	Bandwidth (ω_{BW})
0.01	-0.01	0.013
0.1	-0.1	0.11
1	-1	1.0
10	-10	1.5
100	-100	1.7

As p is reduced, the bandwidth decreases. This leads us to expect slower time response and additional rise time. This effect is significant in case of $p = 0.01, 0.1, 1$, while the bandwidth is hardly changed in case of $p = 10$. Thus, we do not have considerable change in rise time for $p \geq 10$. Bandwidth is a measure of the speed of response of a system, such as rise time.