ECE 486, Fall 2023 **PROBLEM SET 8** **Due Thursday, Oct 26**

Reading: FPE, Sections 6.2, 6.4–6.6, 6.7.1, 6.7.2.

Problems: (you can use MATLAB in all problems, but you must explain all steps and justify all answers)

1. Consider the transfer function $G(s) = \dfrac{1}{s(s^2+4s+8)}$, which already appeared in Problem Set 5.

a) Recall (or rederive) the value of K for which the closed-loop characteristic equation $1 + KG(s)$ has roots on the $j\omega$-axis.

b) For this value of K, make the Bode plot of $KG(s)$ using MATLAB and explain how you can confirm the presence of $j\omega$-axis closed-loop poles using this plot.

c) Compute the gain and phase margins for $K = 12$ using the corresponding Bode plot.

d) Determine the gain K that gives the phase margin of $60°$.

e) Plot the step responses of the closed-loop systems for $K = 12$ and the K you found in part d). Which system has better damping (smaller overshoot)? Why?

2. Consider the transfer function $G(s) = \dfrac{1}{(s-1)(s^2+2s+5)}$.

a) Derive the values of K for which the closed-loop characteristic equation $1 + KG(s)$ has roots on the $j\omega$-axis.

b) For these values of K, make the Bode plots of $KG(s)$ using MATLAB and explain how you can confirm the presence of $j\omega$-axis closed-loop poles using these plots.

c) Compute the gain and phase margins for $K = 7$ using the corresponding Bode plot.

d) What is the largest possible phase margin? Determine the gain K for which it is achieved.

e) The transfer function $KG(j\omega)$ in this problem has a term of the form $(j\omega\tau - 1)^{-1}$ (unstable real pole) which has not been considered in class. Performing an analysis similar to the one done in class for a term of the form $(j\omega\tau + 1)^{-1}$ (stable real pole), explain the contribution of such a term both to the magnitude and to the phase plot.

3. Show that for the transfer function $KG(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s}$, the phase margin is independent of ω_n and is given by

$$PM = \tan^{-1}\left(\frac{2\zeta}{\sqrt{\sqrt{4\zeta^4+1} - 2\zeta^2}}\right).$$

4. Consider the system $G(s) = \dfrac{1}{s(s+1)}$.

a) Design a PD controller that achieves phase margin PM $\approx 90°$ and closed-loop bandwidth $\omega_{\mathrm{BW}} \approx 10$. Verify that the specs are met (be careful: you will need both open-loop and closed-loop data for this).

b) Can you modify the above design to get $\omega_{\mathrm{BW}} \approx 1$, while maintaining PM $\approx 90°$? Explain how or why not.

5. In class we studied the following problem: for the system $G(s) = \dfrac{1}{s^2}$, design a lead controller that gives PM $\approx 90°$ and $\omega_{\mathrm{BW}} \approx 0.5$. This homework problem asks you to check and improve the design given in class.

a) For the controller derived in class:

$$KD(s) = \frac{1}{16}\,\frac{\frac{s}{0.1}+1}{\frac{s}{2}+1}$$

compute the PM, open-loop crossover frequency ω_c, and closed-loop bandwidth ω_{BW}. Plot the closed-loop step response. Explain the reasons why this design didn't fully meet the specs.

b) Improve the design to obtain PM and ω_{BW} closer to the specs. Does the new closed-loop step response show better damping?