ECE 486, Fall 2023 **PROBLEM SET 7** **Due Thursday, Oct 19**

Reading: FPE, Section 6.1.

Problems:

1. Consider the transfer function $G(s) = \dfrac{1}{s^2 + 0.5s + 1}$.

a) Use the formulas given in class:

$$\omega_r = \omega_n \sqrt{1 - 2\zeta^2}, \; M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} - 1 \quad \text{(valid for } \zeta < 1/\sqrt{2}\text{)},$$
$$\omega_{\mathrm{BW}} = \omega_n \sqrt{(1 - 2\zeta^2) + \sqrt{(1 - 2\zeta^2)^2 + 1}}$$

(taken from the book by Kuo, Section 9.2) to compute the resonant frequency ω_r, resonant peak M_r, and bandwidth ω_{BW} for $G(j\omega)$.

b) Use a computer to plot the magnitude $|G(j\omega)|$ as a function of ω (you can use the `bode` or `bodemag` command in MATLAB). Mark the resonant frequency ω_r, resonant peak M_r, and bandwidth ω_{BW} on the graph. Check agreement with the values you computed in a).

2. For each of the transfer functions given below, draw the Bode plots (both magnitude and phase) *by hand*, using the techniques discussed in class. Explain all steps in your drawing procedures. Note that the transfer functions are not given in Bode form.

a) $KG(s) = \dfrac{s + 10}{s(s + 5)}$ b) $KG(s) = \dfrac{8s}{s^2 + 0.2s + 4}$ c) $KG(s) = \dfrac{s^2 + 0.1s + 1}{s(s + 0.2)(s + 4)}$

After you're done, check your results using MATLAB. (Note that the `bode` command in MATLAB plots magnitude in decibels.) Turn in both the hand sketches and the MATLAB plots.

3. Consider the transfer function

$$G(s) = \frac{\frac{s}{a} + 1}{s^2 + s + 1}$$

Use MATLAB to compare the M_p from the step response of the system for $a = 0.01$, 0.1, 1, 10, and 100 with the M_r from the frequency response for the same values of a. Is there a correlation between M_p and M_r?

4. Consider the transfer function

$$G(s) = \frac{1}{(\frac{s}{p} + 1)(s^2 + s + 1)}$$

Use MATLAB to draw the Bode plots for $p = 0.01$, 0.1, 1, 10, and 100. What conclusions can you make about the effect of the pole at $-p$ on the bandwidth compared with the bandwidth for the second-order system without this pole?