ECE 486, Fall 2023 **_LAST_ PROBLEM SET (#12)** **Never due**

NOTE: You don't need to submit this problem set, it is just to help you prepare for the final exam. Solutions will be posted on the web.

Reading: FPE, Sections 7.6 and 7.10.2.

Problems:

1. (exam material) In class we derived the closed-loop system obtained with dynamic output feedback in $(x, \hat{x})$-coordinates:

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A & -BK \\ LC & A - LC - BK \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

and later rewrote it in (x, e)-coordinates. Rewrite the same system in $(\hat{x}, e)$-coordinates.

2. (exam material) Consider the plant transfer function $G(s) = \dfrac{1}{s(s+1)}$.

a) Find any controllable and observable state-space realization of $G(s)$.

b) Stabilize the state-space system from part a) by dynamic output feedback. Select arbitrary controller and observer poles such that the closed-loop system is stable and has reasonable damping (in your judgement).

c) Compute the transfer function of the controller you found in part b). Write it in the form $kD(s)$, where k is a scalar gain (not to be confused with the state feedback gain matrix K) and $D(s)$ is a ratio of monic polynomials (leading coefficients equal 1).

d) Draw the (positive) root locus for $L(s) = D(s)G(s)$ and find on it the locations of the closed-loop poles you chose in part b).

e) Draw the Bode plot for $kD(s)G(s)$ and compute the gain margin and phase margin.

f) Decide whether you're happy with the closed-loop system. If not, go back and improve the design.

3. (not exam material) Consider the system

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2 \\ \dot{x}_2 &= -x_1 + x_2 + u \\ y &= 2x_1 + x_2 \end{aligned}$$

and suppose that the control objective is to minimize the performance index $\int_0^\infty [\rho y^2(t) + u^2(t)]dt$, $\rho > 0$.

a) Show graphically the locations of the optimal closed-loop poles as the parameter ρ varies (symmetric root locus).

b) See why in the limit as $\rho \to 0$ ("expensive control" case), the optimal closed-loop poles become mirror images of the open-loop poles across the imaginary axis.

c) See why in the limit as $\rho \to \infty$ ("cheap control" case), one optimal closed-loop pole cancels the open-loop zero and the other moves off to $-\infty$.