
Reading: FPE, Sections 7.5, 7.7, 7.8, notes on state-space control by Prof. Belabbas (the "legacy documents" section of the course website). Note: some material is developed differently in the book than in class; in these cases, the knowledge of the class approach is mandatory while the knowledge of the book approach is optional.

Problems: (you can use MATLAB to perform necessary matrix computations)

1. Consider the system

$$\dot{x} = \begin{pmatrix} 0 & -1 & 2/3 \\ -1 & -2 & 1 \\ 0 & -3 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} u$$

a) Using the procedure described in class (based on converting to CCF), design a full-state feedback law $u = -Kx$ which places the closed-loop poles at -10 and $-10 \pm 5j$.

b) If the real parts of the desired closed-loop poles were -100 instead of -10, what would happen to the control gains? Give a conceptual answer to this question, without making any extra calculations.

2. Consider the system

$$\dot{x} = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ b \end{pmatrix} u, \qquad y = \begin{pmatrix} 1 & 1 \end{pmatrix} x$$

a) Derive the transfer function using the formula given in class, keeping b a general constant.

b) Show that for $b = 0$, there is a pole/zero cancellation in the transfer function and loss of controllability in the system.

3. Determine (from the observability matrix) whether or not the following systems are observable.

a) $\dot{x} = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix} x, \qquad y = x_2$

b) $\dot{x} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & -5 \\ 3 & 3 & -2 \end{pmatrix} x, \qquad y = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} x$

4. For the system

$$\dot{x} = \begin{pmatrix} 0 & -1 & 2/3 \\ -1 & -2 & 1 \\ 0 & -3 & 1 \end{pmatrix} x, \qquad y = x_2$$

design an observer with observer poles (poles of $A - LC$) placed at -20 and $-20 \pm 2j$. (Follow the procedure described in class, which involves solving the corresponding pole placement problem for an auxiliary system $\dot{x} = Fx + Gu$.)

5. Consider the system

$$\dot{x} = \begin{pmatrix} 0 & -1 & 2/3 \\ -1 & -2 & 1 \\ 0 & -3 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} u, \qquad y = x_2$$

Combine the results of the Problems 1 and 4 to obtain a controller in the form of dynamic output feedback (observer plus estimated state feedback). Write down the state-space model of the controller as well as its transfer function (you can use MATLAB command `ss2tf` to compute the latter).