Please read the following information carefully and start preparing for the exam.

Time and place. The first midterm exam will be held on *Thursday, Sep 28, in class.* It is designed to be a 1-hour exam, so it will start at 12:30pm and you should be able to finish around 1:30pm. There is no conflict exam offered at any other time. Students requiring special accommodations should contact me at this time to discuss specific arrangements.

Topics covered. The exam will cover everything up to and including the pre-recorded Thursday Sep 14 lecture. Here is a list of specific topics:

- Models of simple mechanical and electrical systems; State-space equations; Linearization
- Impulse response and convolution integral; Transfer function, frequency response; Computing transfer functions using Laplace transform, partial fractions method, final-value theorem and DC gain, real poles and transient response
- All-integrator diagrams; Transfer functions for basic block diagrams (series, parallel, and feedback connections) and block diagram reduction
- Prototype 2nd order response, effect of complex poles; Time-domain specifications (rise time, settling time, overshoot) and their interpretation in the s-plane; Effect of zeros; Stability and Routh-Hurwitz criterion
- Open-loop vs. feedback control: reference tracking and disturbance rejection, sensitivity to parameter variations, time response; Basics of PID control, system type

What to bring. The exam is closed-book, closed-notes. You may bring one (double-sided) sheet of notes with any necessary formulas. A calculator will not be necessary or helpful.

Tips for preparing. The primary goal of the exam is to test your understanding of the main concepts, not memorization or computational skills. Make sure to follow up on all lecture material, readings, and homework problems and solutions. You may also find useful the slides by Prof. Max Raginsky posted on the class website. On the next page is an exam from a past semester, solutions to which will be posted later (disclaimer: the exam this semester may be completely different in style and content from that older one). For additional practice, you can look at the problems for Chapters 2–4 in the textbook, but beware that some of them refer to material not covered in class and many of them are much more computationally involved than the problems you will be given on the exam.

Office hours. I will hold extra office hours before the exam, details to be announced. Please also take advantage of homework TAs' office hours to clear up any homework-related questions.

SAMPLE MIDTERM EXAM 1

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1. All questions on this exam refer to the following system, which may describe a spring-mass system under the action of an external force:

$$\ddot{y} + a\dot{y} + y = u \tag{1}$$

Here y is the horizontal displacement of the spring, u is the force, and $a > 0$ is a damping coefficient. (The spring's mass and stiffness coefficient have been normalized to 1.)

a) Convert equation (1) to the state-space form $\dot{x} = Ax + Bu$, $y = Cx$. Make sure to explicitly define the state variables.

b) Calculate the transfer function of the system from u to y. The transfer function should be in the form of the prototype 2nd-order response transfer function

$$\frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Identify the damping ratio ζ and natural frequency ω_n.

c) Assume that $0 < a < 2$. Explain how the choice of a in this range affects the rise time, the overshoot, and the settling time (computed, as usual, with reference to the system's step response).

d) (extra credit) Is it true or false that increasing a beyond $a = 2$ improves the settling time? Justify your answer.

2. Now suppose that, given again the system (1), we want to make the output y track a reference input r by using a feedback controller $u = Ke$, where $e = r - y$ is the tracking error and K is a constant gain.

a) Draw a block diagram of the closed-loop system. (Note: you are *not* asked to draw an all-integrator diagram; the diagram can have the transfer function of system (1) which you found in Problem 1(b) as a single block.)

b) Derive the transfer function of the closed-loop system from r to y.

c) For what values of K is the closed-loop system stable? Justify your answer.

d) Suppose the reference is a constant signal (for example, a unit step, $r(t) = \mathbf{1}(t)$). Explain whether the closed-loop system will achieve perfect tracking ($e(t) \to 0$), imperfect tracking ($e(t) \to \text{const} \neq 0$), or no tracking ($e(t) \to \infty$).

e) Asnwer the same question as in part d) but for the case when the reference is a ramp signal, $r(t) = t\,\mathbf{1}(t)$.

f) Let $a = 1$ and $K = 1$, and let $r(t) = \cos t$. Calculate the steady-state response of the closed-loop system to this input. (As in class, by "steady-state response" we mean the component of the output $y(t)$ that persists after the transients have died down.)