Problem 1. Consider $\dot{x} = \frac{4}{x} - 2u^3$ ($\dot{x} = f(x,u)$ with $f(x,u) = \frac{4}{x} - 2u^3$)

(a) Find an equilibrium point $(\bar{x}, \bar{u})$ with $\bar{u} = 1$

(b) Linearize the system around $(\bar{x}, \bar{u})$.

Solution: (a) set $\dot{x} = 0$, we have $\frac{4}{\bar{x}} - 2\bar{u}^3 = 0$

since $\bar{u} = 1$, we have $\bar{x} = \frac{2}{\bar{u}^3} = 2$

$(\bar{x}, \bar{u}) = (2, 1)$.

(b). $\dot{\delta}_x = \frac{\partial f}{\partial x}(\bar{x}, \bar{u})\delta_x + \frac{\partial f}{\partial u}(\bar{x}, \bar{u})\delta_u$

$\frac{\partial f}{\partial x} = -\frac{4}{x^2} \Rightarrow \frac{\partial f}{\partial x}(\bar{x}, \bar{u}) = -\frac{4}{\bar{x}^2} = -1$

$\frac{\partial f}{\partial u} = -6u^2 \Rightarrow \frac{\partial f}{\partial x}(\bar{x}, \bar{u}) = -6\bar{u}^2 = -6$

$\therefore$ The linearization is $\dot{\delta}_x = -\delta_x - 6\delta_u$.

Problem 2. Consider $\dot{x} = f(x,u)$ with $f(x,u) = -2x^3 + xu$.

(a) Find $(\bar{x}, \bar{u})$ with $\bar{x} = 1$

(b) linearize the system around $(\bar{x}, \bar{u})$.

Solution: (a) $-2\bar{x}^3 + \bar{x}\bar{u} = 0 \Rightarrow \bar{u} = 2\bar{x}^3 = 2 \Rightarrow (\bar{x}, \bar{u}) = (1, 2)$

(b). $\frac{\partial f}{\partial x} = -6x^2 + u \Rightarrow \frac{\partial f}{\partial x}(\bar{x}, \bar{u}) = -6\bar{x}^2 + \bar{u} = -6 + 2 = -4$

$\frac{\partial f}{\partial u} = x \Rightarrow \frac{\partial f}{\partial u}(\bar{x}, \bar{u}) = \bar{x} = 1$

The linearization is $\dot{\delta}_x = -4\delta_x + \delta_u$

Problem 3. Consider $\dot{x} = f(x,u)$ with $f(x,u) = -4x - 2u + u^3$

(a) Find $(\bar{x}, \bar{u})$ with $\bar{u} = 2$

(b) Linearize around $(\bar{x}, \bar{u})$.

Solution: (a) $-4\bar{x} - 2\bar{u} + \bar{u}^3 = 0 \Rightarrow \bar{x} = 1 \Rightarrow (\bar{x}, \bar{u}) = (1, 2)$

(b) $\frac{\partial f}{\partial x} = -4 \Rightarrow \frac{\partial f}{\partial x}(\bar{x}, \bar{u}) = -4$

$\frac{\partial f}{\partial u} = -2 + 3u^2 \Rightarrow \frac{\partial f}{\partial x}(\bar{x}, \bar{u}) = -2 + 3\bar{u}^2 = 10$

$\therefore \quad \dot{\delta}_x = -4\delta_x + 10\delta_u$