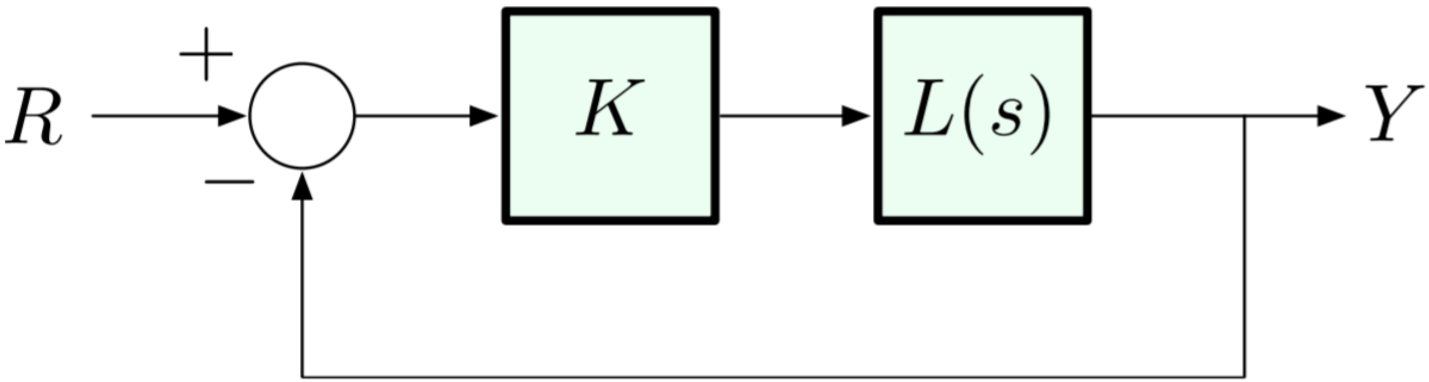
ECE 486: Control Systems

Lecture 11A: Introduction to Root Locus Method

Problem 1

Suppose $L = \frac{1}{s^2+2s}$.



(a) Solve the closed-loop poles as a function of K.

(b) Draw the root locus.

(c) Is it possible to select K to achieve settling time $\leq 3s$?

Solution 1A

- Solve the closed-loop poles as a function of K

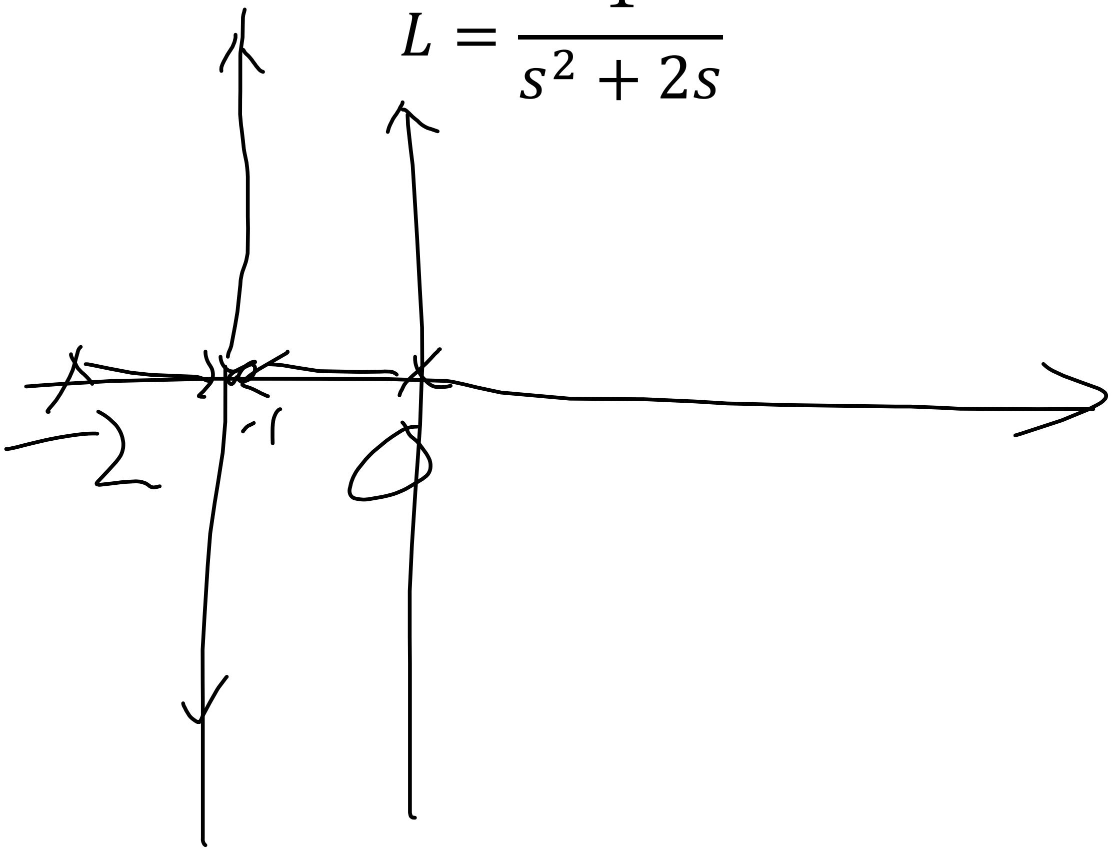
$$L = \frac{1}{s^2 + 2s}$$

Answer: $s^2 + 2s + K = 0 \Rightarrow \mathrm{s} = -1 \pm \sqrt{1 - K}$

Solution 1B

- Draw the root locus.

$$L = \frac{1}{s^2 + 2s}$$



Solution 1C

- Is it possible to select K to achieve settling time ≤ 3 seconds?

$$L = \frac{1}{s^2 + 2s}$$

Answer: Yes, choosing K=1 leads to a time constant ≈ 1 second and a settling time ≈ 3 seconds

ECE 486: Control Systems

Lecture 11B: Root Locus Rules ABC

Problem 2

Consider the following functions.

$$L = \frac{1}{s^2+2s+10}, \qquad L = \frac{s-3}{s^2+2s+10}, \qquad L = \frac{s+4}{s^5+1}$$

How many branches are there in the root locus? What are the starting and ending points? Justify your answers using Matlab.

Problem 2A

Consider the following functions.

$$L = \frac{1}{s^2+2s+10}$$

How many branches are there in the root locus? What are the starting and ending points? Justify your answers using Matlab.

Answer: Two branches. Starting points: $-1 \pm 3j$

Ending points: two points at infinity

Problem 2B

Consider the following functions.

$$L = \frac{s-3}{s^2 + 2s + 10}$$

How many branches are there in the root locus? What are the starting and ending points? Justify your answers using Matlab.

Answer: Two branches. Starting points: $-1 \pm 3j$

Ending points: 3 and another point at infinity

Problem 2C

Consider the following functions.

$$L = \frac{s+4}{s^5+1}$$

How many branches are there in the root locus? What are the starting and ending points? Justify your answers using Matlab.

Answer: Five branches. Starting points are the solutions for $s^5 + 1 = 0$. We can numerically solve the solutions as -1, -0.31 $\pm$ 0.95j, 0.81 $\pm$ 0.59j.

Ending points: One point at -4 and the other four points are at infinity