Reading: FPE, Sections 5.1, 5.2, 5.6.1, 5.3, 5.4.1, 5.4.2, 5.5.

Problems:

1. Consider the plant with transfer function \( L(s) = \frac{1}{s^2 + 2s} \). Under the action of a constant feedback gain \( K \), the closed-loop poles are the roots of the characteristic polynomial \( s^2 + 2s + K \).
   a) Draw the (positive) root locus. (Use the expression for the closed-loop poles in terms of \( K \) obtained via the quadratic formula.)

   b) Consider the settling time spec \( t_s \leq 4 \). Give some value (or range of values) of \( K \) for which the closed-loop system meets this spec. Justify your choice. Show the corresponding pole locations on the root locus.

   c) Consider the rise time spec \( t_r \leq 1 \). Give some value (or range of values) of \( K \) for which the closed-loop system meets this spec. Justify your choice. Show the corresponding pole locations on the root locus.

   d) Consider the overshoot spec \( M_p \leq 0.1 \). Give some value (or range of values) of \( K \) for which the closed-loop system meets this spec. Justify your choice. Show the corresponding pole locations on the root locus.

   e) Suppose that it is desired to place the closed-loop poles at \( -1 \pm j \). Find the value of \( K \) that will achieve this, using the characteristic equation \( s^2 + 2s + K = 0 \) but without using the quadratic formula. (In other words, you should find a way of doing this that would also work for a higher-order example.)

2. Consider the following transfer functions:
   \[
   1) \quad L(s) = \frac{1}{s(s^2 + 4s + 8)} \quad \quad 2) \quad L(s) = \frac{s}{(s - 1)(s + 1)^3}
   \]
   For each one of these, do the following:

   a) Mark the zeros and poles on the \( s \)-plane and use Rule 2 from class to plot the real-axis part of the root locus.

   b) Use the phase condition from class to test whether or not the point \( s = j \) is on the root locus. If you run into “non-obvious” angles, estimate rather than calculate them, this should be enough.

   c) Apply Rules 3 and 4 to determine asymptotes and departure and arrival angles. Plot the root locus branches based on this information.

   d) Apply Rule 5 to determine imaginary-axis crossings (if any), and complete the (positive) root locus by using Rule 6 to check for multiple roots.

   e) Plot the (positive) root locus using the MATLAB \texttt{rlocus} command.

Turn in your MATLAB plots as well as hand sketches of root loci along with all accompanying calculations and explanations.

3. Consider the transfer function \( L(s) = \frac{s^2 + 2s + 2}{s^2 - 2s + 2} \).
   a) Plot by hand the negative \( (K < 0) \) root locus for \( L(s) \), using Rules 1–6 for negative root loci. Make your root locus as explicit as possible by specifying (when applicable) the real-axis part, asymptotes, arrival and departure angles, imaginary-axis crossings, and points of multiple roots. Turn in the hand plot and accompanying calculations and explanations.

   b) Plot the same root locus in MATLAB. Turn in the MATLAB plot.